

Abstract

Investigation and analysis of the Gibbs House located in Kalamazoo, Michigan revealed the many, and often, ubiquitous complexities prevalent in historic residential homes. The core issues relate to insulation, fenestration thermal behavior, building envelope construction, airtightness, and energy systems performance. In an effort to improve occupant comfort and building energy performance, a whole systems approach was employed to determine feasibility and necessary steps to meet the rigorous Passive House retrofit standard of annual total heating and source energy below 7.9 kBtu/ft² and 38 kBtu/ft², respectively. Additionally, solutions are presented that aim to maintain the historic integrity of the 154 year old home. Deficiencies in each of the five Passive House target areas of insulation, fenestration, building envelope, airtightness, and energy systems were observed throughout many field investigations, occupant interviews, infrared thermography sessions, finite element analyses, blower door tests, and energy simulations. The insulation analysis revealed deteriorated and nearly non-existent mineral wool insulation within the wall cavity. Original single pane glass windows have loose framework, broken panes, worn caulk, and a U-factor of 0.687 Btu/h-ft-F compared to new double pane U-factors below 0.31 Btu/h-ft-F. The balloon framing is prone to thermal bridging and high heat flows specifically through air cavities and exterior siding. Infiltration occurs at multiple points across the building envelope including the attic, basement, and floor joists. Window framework is prone to air change rates of 2.8 ACH50 compared to the Passive House standard of 0.6 ACH50. Mechanical and plumbing equipment is out of date by eight years, lighting is non-LED with no occupancy sensors, and no mechanical ventilation is provided to occupants. Overall, the building was observed to not meet Passive House retrofit standards, as well as Michigan Building Code in some instances, in each target area. Facility improvement measures were recommended to target each deficiency based on energy simulation results, payback time, and positive impact on residents. Initial action recommendations include replacing mechanical and plumbing equipment and provide piping insulation, replacing lighting with warm hue LEDs, installing an attic fan, and room ceiling fans; repairing all broken window panes and frames; weatherstripping all windows and installing film during the heating season; blow-in new cellulose insulation via baseboard penetrations for exterior walls; and provide basement ribbon insulation and air sealing. These initial action recommendations are expected to require approximately \$38,800 in capital cost with \$1,800 annual energy savings, resulting in a twenty two year payback time.

Keywords: historical residential retrofit, facility improvement measures, finite element analysis, building envelope, passive house, energy modeling.

Gibbs Passive House Study:
A case study approach for a historic residential
energy efficiency renovation

by

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Chapter 1

1 Introduction

Residential buildings rely on approximately 19% of the primary energy consumed in the United States [1]. Many of these buildings are designed and built according to minimum building code rather than energy conscious standards such as Passive House. Additionally, 17% of all homes in the United States were built before 1950 and 30% of all homes in Michigan were built prior to 1950 [1]. The need for improving the housing stock is even more apparent as these pre-existing buildings are prone to experience deterioration over time of building envelope performance. The building envelope is composed of the walls, insulation, roof, floor, and openings in order to provide a barrier between the interior and exterior of the building. Therefore, deficiencies in the building envelope result in reduced energy efficiency and poor thermal comfort due to undesirable heat transfer through walls and openings and inadequate humidity levels [2], [3]. Examples of building envelope deficiencies are low R-value insulation, conductive thermal bridge materials, high U-factors for windows, and permeable or poorly located vapor barriers. Typically, techniques to counteract deficiencies in the building envelope are employed based on industry standards rather than individual in-depth analyses [4],[5]. “Homeowners, designers, and those in the building trades have few tools to evaluate how various strategies for retrofitting of existing windows compare to replacing them with new windows” [6]. Although industry standard methods may save time and money on the front end, a more targeted approach to appropriately diagnose the underperformance issues may result in increased building envelope and energy performance over the continued life of the building. Therefore, a complete building investigation and proposal covering insulation, fenestration, building envelope, airtightness, and energy systems was conducted for an historical building on campus. The objective of the research is to determine innovative approaches to convert the Western Michigan University Gibbs House into a passive house while meeting occupant thermal comfort levels and rigorous historic preservation guidelines. Unique, but scalable, methods are recommended in order to bring the building closer to meeting an annual total heating energy below 7.9 kBtu/ft² and an annual total source energy below 38 kBtu/ft² [7], [8] while maintaining preservation constraints to honor the historic integrity of the building. This study aims to both address a common misconception that old buildings cannot be retrofitted cost effectively and challenge the norm of designing buildings just to meet code.

1.1 Context

The Gibbs House is a historic farmhouse residence built in 1853 and owned and operated by Western Michigan University. The house is located on Parkview Avenue in Kalamazoo, Michigan. The house is a functioning residence with three to five students living in the Gibbs House every year as fellows for the WMU Office for Sustainability. The house is located on a two acre permaculture demonstration and research site.



Figure 1: Gibbs House pictured (left); Revit model (right)

1.2 Selection of Performance Rating Method

Passive House is a building standard that focuses on energy efficiency, occupant comfort, and affordability. In order to achieve a Passive House rating, a building must comply with five core building science principles: continuous insulation, airtight envelope, high performance windows, minimalistic heating and cooling systems, and passive heating and cooling measures. Passive House retrofit standards follow the EnerPHit program which allows for less stringent requirements for older homes: 7.9 kBtu/ft²/year for heating and cooling, 1 ACH at 50Pa for airtightness, and 38 kBtu/ft²/year for total energy usage [8]. The Gibbs House passive house study delves into the EnerPHit methods and metrics.

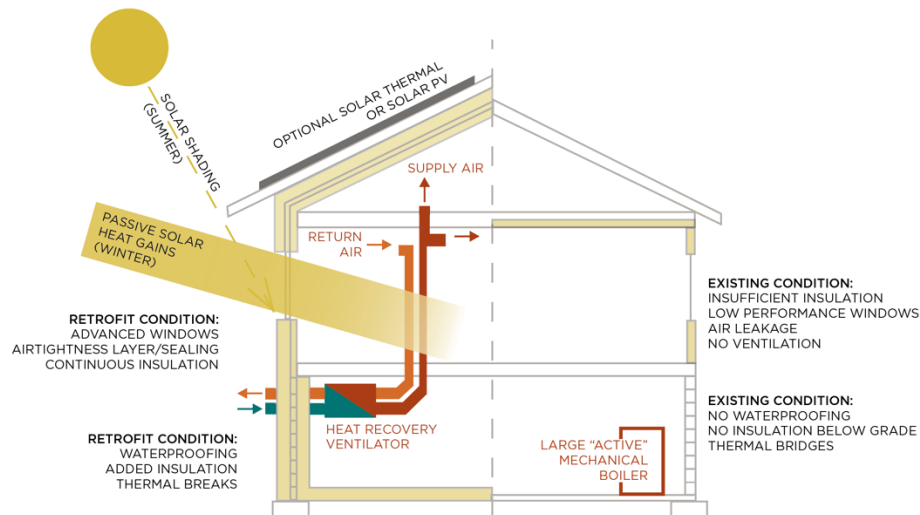


Figure 2: Passive House holistic approach

In addition to Passive House standards, the study considers regulatory agencies that provide constraints, restrictions, and guidelines for historic renovations in order to preserve the integrity of culturally rich homes. Federally, the Secretary of the Interior provides information for the treatment of historic homes and the National Trust for Historic Preservation quantifies the environmental value of building reuse [9]. The State of Michigan, the City of Kalamazoo, and Western Michigan University each govern renovation endeavors [10], [11].

1.3 Objectives and Thesis Summary

The overarching goal of this research is to provide detailed information for the building owner through robust energy modeling and customized, cost effective facility improvement measures. The study serves as a proposal for future renovation work at the Gibbs House and as a guide for the building owner when reviewing suggested recommendations.

For the building itself, the objectives are to increase occupant thermal comfort and controllability of systems, improve indoor air quality, decrease energy consumption through system efficiency measures, and increased building performance.

The objective of this research is to focus on each building element that contributes to energy consumption or occupant discomfort and propose cost effective solutions. This process involves testing the existing conditions, implementing targeted and temporary improvements to be tested for efficacy, modeling energy consumption on a whole building scale, determining realistic cost estimates for future work, and calculating return on investment periods due to energy savings. The extent of this process is detailed in the Methodology section.

The philosophical intent of this research is to demonstrate how a thorough, building science based investigation into the specific challenge and opportunities of a particular historic building can move well beyond turnkey solutions and result in a higher performing, more cost effective retrofit.

Chapter 2

2 Literature Review

The Secretary of the Interior maintains a National Register of Historic Places in addition to developing guidelines for preservation, rehabilitation, restoration, and reconstruction of these places. The guidelines provide recommendations for all phases of a historic project relating to sustainability and energy efficiency. Table 1 highlights the recommendations relevant to the Gibbs Passive House Study.

Table 1: Secretary of the Interior Rehabilitation recommendations [9]

Category	Relevant Recommendations
Planning	Identify ways to reduce energy use prior to considering invasive strategies
Maintenance	Maintain historic buildings regularly; retain and repair durable historic building materials; use environmentally friendly cleaning products
Windows	Retain and repair historic windows when deteriorating; weatherstrip and caulk historic windows to make them weather tight; Install interior or exterior storm windows that are compatible with existing historic windows; retrofit windows with high performance glazing if possible
Weatherization	Utilize blower door, infrared thermography and energy modeling to gain understanding of the building's performance and potential; eliminate infiltration first using the least invasive measures to start (i.e. weatherstripping)
Insulation	Insulate unfinished spaces, such as attics, basements, and crawl spaces first; ensure air infiltration is reduced before adding wall insulation
HVAC Systems	Retain and maintain functional HVAC systems; upgrade systems to increase efficiency while maintaining historic character of the building; supplement the HVAC systems with programmable thermostats, attic and ceiling fans, and vents; consider ductless air conditioners rather than ducted, central air conditioning systems

The Preservation Green Lab developed a study focused on comparing window retrofit and replacement options for older homes. The “results of this analysis demonstrate that a number of existing window retrofit strategies come very close to the energy performance of high-performance replacement windows at a fraction of the cost” [6]. This study challenges the common misconception that all old windows must be replaced.

The Minneapolis MinnePHit House resembles the intent of the Gibbs House renovation. The pseudo-Tudor style home built in 1935 is approximately 1200 square feet with five residents. The goal for renovation was to reduce heating loads by 80%, reduce overall energy used by two-thirds, and reduce air

leakage rates by ten times to meet the Passive House EnerPHit standard for retrofits. They recognized that human behavior change would be necessary in order to decrease building energy due to usage patterns. Initially, infrared thermography images were taken of the single pane windows, piping, vents, and the entire building. The heating plant was found to have difficulty reaching temperatures above 64F. Additionally, the building was blower door tested at 8.5 ACH50, or 2,100 CFM at 50 Pa. No mechanical ventilation was available in the building, daylighting was not provided, and thermal comfort was determined to be undesirable. The team set up a holistic approach to achieve code compliance and the EnerPHit standard by adding temperature controls, ventilation, a tighter envelope, continuous insulation, while overlaying the architectural program and aesthetic. Spray foam and vacuum insulated panels were installed on the interior and dense packing cellulose was installed from the exterior for additional building envelope insulation. After modifications were made, including a new energy recovery unit and controls, the building blower door tested at 0.87 ACH50 and 267 CFM at 50 Pa. The overall cost for the retrofit was approximately \$300,000. [12]

The ReNEWW House operated by Purdue University and Whirlpool Corporation is another older home renovated for energy efficiency and education in a similar climate zone to Kalamazoo, Michigan. During a site visit to the home, the building was found to be a highly technological house retrofitted with an energy recovery unit, rainwater collection and a custom filtration system, hyper-controllability and data logging even down to residents' heat rates, and highly efficient and state of the art appliances and equipment. Mechanical ventilation is provided by an energy recovery unit and heating hot water is provided by a deep geothermal well. Data is available on their website to reveal, for example, that HVAC energy consumption was reduced by approximately 80% after the renovation [13].

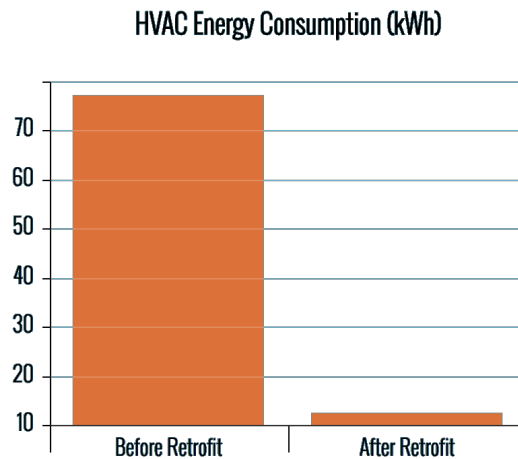


Figure 3: ReNEWW House HVAC energy consumption data [67]

The heating and cooling demand is strongly related to the window-to-wall ratio (WWR), often because the window u-factor and solar heat gain coefficient is much higher than that of the wall material [14]. The net glazing area, A_g , divided by the gross wall area, A_w , determines the WWR, as shown by Equation 1.

$$WWR = A_g / A_w \quad \text{Equation 1}$$

Figure 4 demonstrates two core concepts relating to energy load and building design. First, a wall that is oriented south will receive more direct sunlight, and thus increase heat gain from solar radiation, resulting in a lower heating demand. The building under analysis in Figure 4 is located in South Korea at 37.3 degrees latitude. This latitude is similar to the latitude of the Gibbs House in Kalamazoo, Michigan (47.3 deg N) and thus yields similar thermal behavior according to sun paths in the northern hemisphere. Second, as the window-to-wall ratio increases noting that the U-factor is larger for windows than walls, the heat loss in winter from transmission and the heat gain from solar radiation both increase [15].

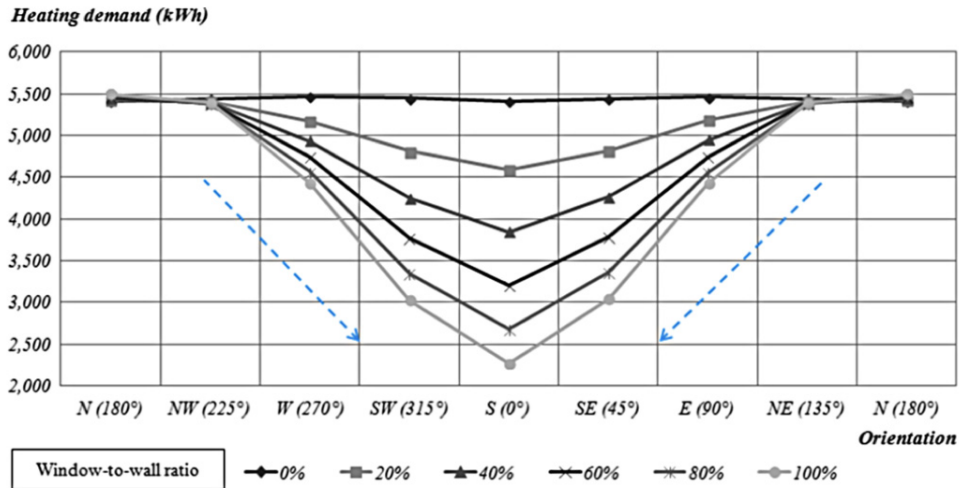


Figure 4: Heating demand curves [15]

A higher performing window, as well as a lower window-to-wall ratio, results in a decreased impact on the overall building energy use.

Chapter 3

3 Methodology

A five tiered approach is utilized to develop an understanding of the building’s energy and architectural performance based on the Passive House performance model. Each of the five target areas have a unique or industry standard technique that allows for in-depth analysis. The novel approach stems from implementing all five strategies in order to develop a robust understanding of the thermal behavior and energy performance of the building at multiple scales. Throughout the research process, facility improvement measures were identified to target each of the research areas. The origin, data, and explanations for each measure are provided in this section as well as the Results section. However, the organizational categories are shown in Table 2.

Table 2: Alternative descriptions

Alt Category	Alternative	Description
Alt 1	As-Is	Existing Gibbs House condition
Alt 2	MEP Upgrade	New electric boiler & water heater, new finned tubes, new attic fan & ceiling fans, LED lighting replacement
Alt 3	Windows Only	Building as-is, except updating window options
Alt 4	New envelope	New building envelope with 4" added exterior insulation, new siding to match historical aesthetic, new double pane windows
Alt 5	Deep retrofit	MEP upgrade (Alt 2), new double pane windows (Alt 3E), and new building envelope (Alt 4)

The categories correspond to a Passive House target measure in order to improve the building energy performance. The numbering represents the alternatives described above to denote the scope of testing conducted. The letters corresponding to fenestration relate to the five different window alternatives tested and compared.

Table 3: Facility improvement categories

Category	Related Alternatives
Insulation	Alt 4, Alt 5
Fenestration	Alt 3 (A,B,C,D,E)
Building Envelope	Alt 4, Alt 5
Airtightness	Alt 3, Alt 4, Alt 5
Energy Systems	Alt 2, Alt 5

3.1 Insulation Analysis

Performance of existing insulation was analyzed using infrared thermography, endoscopy, and steady state thermal circuit calculations.

3.1.1 Infrared Thermography

Thermography is a noninvasive process that utilizes the infrared spectral band of light to record temperature images [16]. Real time temperature measurements can be found using an infrared thermographic camera. Typically, thermographic scans are used to indicate the effectiveness of insulation based on temperature ranges and anomalies. The acquisition of temperature data was conducted using a ThermaCAM E65 infrared camera from FLIR Systems. Technical specifications are shown in Table 4.

Table 4: Technical specifications of FLIR ThermaCAM E65 [16]

Field of View (FOV) focus distance	Spectral Range	Detector Type	Accuracy	Thermal Sensitivity
15" x 19" x 11.8"	7.5 to 14 μm	Focal plane array (FPA) uncooled microbolometer	+/-2 degC	0.12 degC

The U-factor (Btu/hr-ft²-degF) is a measure of the rate of heat transfer through a window and indicates insulation performance [17]. Lower U-factors indicate a greater resistance to heat flow and therefore have greater insulative value [6]. Infrared thermography was used to measure actual temperatures which were used to calculate the U-factors for the walls and window glass as a reference point. The glass U-factor was then used in THERM [18] to calculate the total window assembly U-factors. Equation 2 provides a means for solving for the U-factor using empirical data. The U-factor depends on: indoor environment temperature (T_{int}), outdoor environment temperature (T_{out}), surface temperature (T_w), emissivity on the entire spectrum (ϵ_{out}), and wind velocity (v). Wind velocity was assumed to be 0.5 mph based on local weather data on the relatively calm day measurements were taken for this calculation. The emissivity was assumed to be 0.8 based on the FLIR emissivity table and the white surface paint on the building façade and interior plaster.

$$U \text{ factor} = \frac{5.67\epsilon_{\text{tot}}\left[\left(\frac{T_w}{100}\right)^4 - \left(\frac{T_{\text{out}}}{100}\right)^4\right] + 3.8054v(T_w - T_{\text{out}})}{(T_{\text{int}} - T_{\text{out}})} \quad \text{Equation 2 [19]}$$

Infrared thermography (IRT) images were taken in order to observe the temperature distribution across the building envelope. The cold weather images were gathered specifically to observe envelope performance when subjected to extreme temperatures in the worst case condition. Since the IR images only display surface temperatures, images were taken on both the interior and exterior of the building in order to observe a general temperature change across the envelope.

3.1.2 Wall Assembly Endoscopy

A minimally invasive approach was utilized to observe the present state of insulation within the wall cavity using an endoscope. Since no major renovations have been made to the interior of the building envelope since original construction, the level of insulation was unknown. Therefore, an endoscope was utilized at multiple locations throughout the building to visually inspect the percentage of insulation volume within the wall cavity.

3.2 Fenestration Thermal Performance Analysis

The fenestration analysis is setup as a series of alternatives to demonstrate possible window improvement measures based on thermal and infiltration performance. Refer to Table 5 for brief descriptions of each proposed alternative.

Table 5: Window alternate descriptions

Alt.	Name	Description
A	Repairs	Repair broken panes and framework, install missing latches (leave in place)
B	Weatherstrip	Weatherstrip sash and side cavities
C	Weatherstrip & film	Weather per Alt B and install film during heating season
D	Refurbished	Remove and upgrade windows by re-caulking and replacing panes
E	New double pane	Install new high performance double pane windows
F	Storm Windows	Install or replace exterior storm windows

3.2.1 Finite Element Approach

Finite element method (FEA) is a numerical method for approximating solutions using discrete portions and meshing [20]. The finite element analysis portion of the study focuses on a three stage approach involving a two-dimensional heat transfer finite element analysis that utilizes conduction and linearized convection and radiation behavior. The FEA model is incorporated into the workflow to calculate complete window assembly U-factors used in the Trace 700 whole building energy model.

Infrared temperature grid data revealed high levels of heat loss through existing windows. Typically, “Since the U-value and solar heat gain coefficient of the window [are] higher than those of the wall, as the window-to-wall ratio increased, both the heat loss due to heat transmission in winter and the heat gain due to the effect of the solar radiation were increased” [15]. This phenomenon was demonstrated

graphically and numerically in the finite element cross sections that show heat transfer through the assemblies.

Certain parameters that affect building heating and cooling demand were assumed constant for this analysis to simplify the shape function and serve as a case study for the building under investigation. These parameters include region (climate zone 5A) and location. Location is considered constant because the study focuses on one building in a fixed location. However, input values can easily be updated for future buildings of similar type. The orientation of each wall is taken into consideration for the energy calculations and summed for the whole building analysis.

The finite element model was performed on an individual window scale in THERM, an open source software program provided by the Lawrence Berkeley National Laboratory (LBNL) [21],[22],[23]. The model assumes two dimensional steady state heat transfer. Two dimensions were determined to be sufficient since most “fully three-dimensional heat transfer simulations require complex methods for describing the model geometry. This added complexity is usually not justified by the modest increase in accuracy for most applied problems in buildings” [24]. Conduction and radiation heat transfer are computed explicitly. Natural convection is modeled within cavities using correlations and convection boundary conditions with standard and custom heat transfer coefficients [22].

Boundary conditions are applied on the glazing and frame surfaces exposed to interior and exterior air. The top of the glazing system and the bottom of the frame are assumed adiabatic. “Boundary conditions on indoor and outdoor surfaces consist of both a convection and radiation component. The convection component on the indoor side is specified through the use of a temperature dependent surface heat transfer coefficient, based on natural convection correlations. For each frame material type there is a constant value of the convective surface heat transfer coefficient. The radiation component is modeled explicitly through the use of a detailed, view-factor-based radiation model. This model assumes that the indoor environment has a uniform temperature and the emissivity of a blackbody” [22]. The mesh control for the QUADTREE is set to 8, which results in at least two elements per geometry thickness. In some instances, such as for the single pane whole assembly setup, there may only be one element per geometry thickness in a given area. This lends itself to possible error or inaccuracy in results. However, since U-factors are being calculated on a whole window assembly scale, the error is deemed negligible with respect to the entire assembly. Additionally, these instances are typically small in scale relative to the entire assembly. The error estimator has a maximum iteration of 5 with 10% maximum error based on the energy equation. The convergence tolerance is 1E-6. After the mesh sizing is determined by the quadtree, the algorithm finalizes well-shaped quadrilaterals and triangles. All elements are treated as quadrilaterals for simplicity where a triangle is defined as a quad with two coincident nodes. The finite element solving method is based on the method of weighted residuals with the form:

$$\iint_{\Omega} W \mathfrak{R} dx dy = 0 \quad \text{Equation 3 [25]}$$

The residual function, \mathfrak{R} , occurs when the actual temperature field is approximated and plugged into Equation 4. The exact temperature field results in a zero residual. The weighting factor, W , forces the

integral to zero and minimizes error of the solution. CONRAD is the algorithm behind THERM and is based on the Galerkin form of the weighted residual method using algebraic shape functions that are equal to the weighting function. Equation 4 is solved for each element then approximated using a numerical integration for second order Gaussian quadrature.

$$k \iint_{\Omega} \left(\frac{\partial W}{\partial x} \frac{\partial \theta}{\partial x} + \frac{\partial W}{\partial y} \frac{\partial \theta}{\partial y} - W q_g \right) dx dy + \int_{\Gamma} W h (\theta - \theta_{\infty}) d\Gamma - \int_{\Gamma} W \bar{q} \Gamma = 0 \quad \text{Equation 4 [25]}$$

After the numerical simulation is calculated, the error estimator makes local error estimates and then refines troublesome regions of the model and recalculates the entire model. According to the THERM NFRC Simulation Model [REF], this procedure is repeated until no local regions show error levels higher than what is prescribed. The error level utilized in this study is a maximum of 10% nominal energy error with a maximum of five iterations. The convergence tolerance set by the CONRAD simulation algorithm is 1E-6 as a default. The iterative model calculates the nonlinear temperature relation that develops in combined conduction and radiation simulations.

The thermal behavior across the window assembly was simplified and studied by conducting a 2D thermal analysis across a window assembly section. The discretization of both the framing section and window section were done simultaneously in THERM. The heat flux and U-factor due to window and framing type was the output to be calculated based on material properties and temperature boundary conditions. Numerical integration was used to solve the differential equation via Gauss Quadrature for 2-point integration [26], [27]. The general governing equation includes convection and linearized convection as shown in Equation 5 [28], [29].

$$\{\partial\}^T ([K]\{T_{\partial}\}) + c\rho \frac{\partial T}{\partial t} = 0 \quad \text{Equation 5 [26]}$$

The applied boundary condition, in this case convection, can be defined as a flux:

$$f_b = \{u\}^T [K]\{T_{\partial}\} \quad \text{where} \quad f_b = h(T_{fl} - T_s) \quad \text{Equation 6 [26]}$$

Window section geometries were drawn in Revit for each of the proposed window alternatives: existing as-is single pane, repaired single pane, weatherstripped single pane, weatherstripped and film single pane, refurbished single pane, and new double pane windows. The double pane geometry was provided by the manufacturer and existing single pane geometry was based on the previously mentioned assumed wall construction.

Finite element results are utilized in Trane Trace 700 software to conduct whole building energy consumption calculations as if the window under consideration were installed. Trace runs calculations based on weather data, construction materials, mechanical, electrical and plumbing systems, and occupancy patterns.

3.3 Building Envelope Analysis

The Gibbs House was built in 1853 on Parkview Avenue in Kalamazoo, Michigan. Since the analysis is based on an existing historic building, little information was initially known about the wall construction and assembly type. Balloon construction was common from the mid-1800s to 1950s [30]. Refer to the Insulation section for details on building envelope assumptions. Material properties of the windows were assumed since the original building documents were not available. The known material properties and dimensions are shown in Tables 6 and 7. Asbestos and lead testing was conducted by WMU Environmental Health and Safety for the specific areas that may be impacted during renovation recommendations.

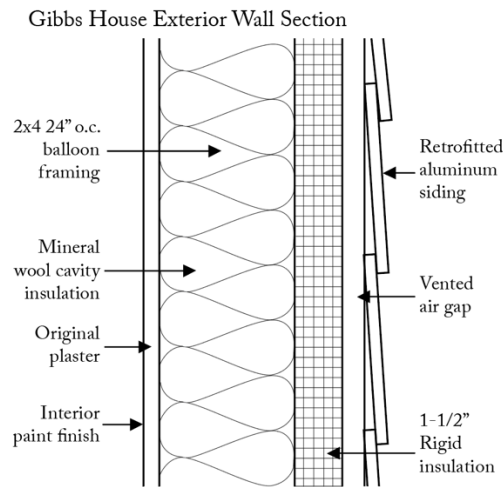


Figure 5: Gibbs exterior wall section

Table 6: Building envelope properties

	Description	Surface Area, A (SF)	Thickness, L (ft)
Wall Assembly	Wood balloon 2x4 framing, retrofitted aluminum siding, interior plaster finish	2,924	0.667
Windows	Double hung, single pane wood framing	479	0.083
Door	Existing wood door and framing	60	0.1663
Floor	Wood and wood subfloor	1,110	0.832
Ceiling/Roof	Retrofitted insulative batting above ceiling	1,110	0.832

Based on the materials present in the wall assembly, the following thermal conductivity properties are assigned based on ideal conditions. Actual insulative properties are determined later in the study.

Table 7: Wall assembly layer properties

	Description	Thickness (ft)	Thermal Conductivity (Btu/hr-ft-F)*
Plaster	Existing 1" historic plaster and finish	0.0833	0.25
Cavity Insulation	Existing 4" mineral wool	0.33	0.00676
Rigid Insulation	Assumed R-4, 1-1/2" rigid insulation	0.125	0.00367
Air gap	1/2" air gap	0.04165	0.0149
Siding	Retrofitted 1" aluminum siding	0.0833	121

*Based on [59]. Actual insulative properties demonstrated in Results section.

3.3.1 Two Dimensional Thermal Conduction

Heat conduction through the building envelope is assumed to be steady state, with constant discrete thermal conductivities and no internal heat sources. Therefore, the Laplace equation may be utilized as a special case of the general Fourier heat conduction equation for the building envelope assembly in two dimensions.

$$\nabla^2 T = 0 \quad \text{Equation 7 [31]}$$

The differential equation is solved by separation of variables of the form:

$$T = XY \quad \text{where } X = X(x) \text{ and } Y = Y(y) \quad \text{Equation 8 [31]}$$

Boundary conditions are then applied to find the temperature distribution across the wall section using a separation constant, γ . The solution for the Laplace differential equation is found using the Sturm-Liouville method as shown in the Results section.

3.4 Airtightness Analysis

Airtightness of the Gibbs House was analyzed using a blower door depressurization test. The Minneapolis Blower Door by The Energy Conservatory [32] was setup using a single fan Model 3 System. Many iterations of a “one-point” test were conducted at 50 Pa to determine the air leakage within the building for the whole building and for each window alternative. The blower door setup comprised of a calibrated fan sealed within an exterior doorway that forces air out of the building to depressurize interior spaces. Air is then drawn naturally into the building through cracks, air gaps, leaks, and other deficiencies in the building.

Whole building leakage rates were determined by installing the blower door in the front door, closing all windows, and covering all vents. Tests were conducted for the building as-is, with temporary minor repairs, and custom weatherstripping. The whole building test takes into consideration not just the anticipated fenestration leaks, but all other possible leakage points such as the basement ribbon, chimney, or other envelope leakage points.



Figure 6: Blower door installed for whole building (left); example screen showing 50Pa building pressure (right)

The standalone window tests were conducted to observe leakage rates for the window alternatives on a single room basis. Infiltration data from the blower door, in conjunction with fenestration thermal behavior observed in the other section, is intended to provide a more complete profile for determining the best window upgrade option. The fan controller maintains the building pressure at approximately 50 Pa and airflow reading samples are provided at every two seconds for 1-2 minute intervals after the fan maintained 50 Pa.



Figure 7: Standalone window baseline (left); standalone retrofitted window installed (right)

3.5 Energy System Analysis

The energy systems within the Gibb House and their respective efficiencies contribute to the overall energy use of the building. In order to reduce energy consumption and preserve the life of the building and its equipment, an investigation of the building “core” was conducted for mechanical, electrical, and plumbing systems. Thermal comfort is also investigated as it relates to heating, ventilation, and air conditioning of interior spaces.

3.5.1 Mechanical, Electrical, and Plumbing Systems Analysis

The Gibbs House does not utilize space cooling or mechanical ventilation. The space heating is provided by hot water perimeter fin radiators installed during the 1970’s. Domestic hot water is provided by a natural gas tank water heater located in the basement. Direct exhaust to the exterior is provided in the kitchen and bathrooms.

Table 8: Mechanical system specifications

Domestic Hot Water	
Manufacturer & Model	AO Smith FGC4246
Capacity	40 gal
Max working pressure	150 psi
Foam Insulation	R-7
Circulation Pump	B&G NRF-22
Liquid Temperature	240F
Year Installed	1994 est.
Heating Hot Water	
Manufacturer & Model	Weil McLain
Input Capacity	140 kBtu/hr
AFUE	0.79
Year Installed	1988
Piping	Uninsulated

Initial field investigations revealed the natural gas boiler is 29 years old with uninsulated piping. According to ASHRAE, gas fired furnaces are expected to operate as designed for a life expectancy of 18 years [33]. The domestic hot water heater age is approximately 23 years, however the average life expectancy is 15 years. Therefore, updated mechanical systems are considered throughout the study. Updated lighting is also considered to be replaced with residential type warm hue LEDs as part of the mechanical upgrade option.

Energy modeling is conducted using Trane Trace 700 software [34]. Trace 700 is a common industry standard for whole building energy modeling based on ASHRAE 90.1-2007 [35] that accounts for weather data, building construction properties, equipment type and sizing, systems configurations, and economic information. The software is built to calculate comparisons across alternatives that allow for the multi-faceted alternative approach developed in this study.

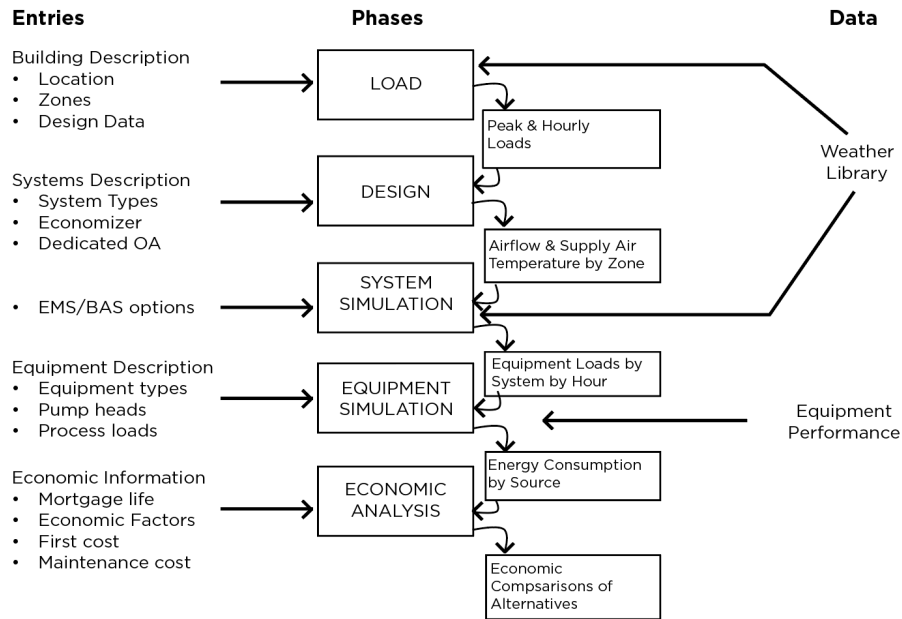


Figure 8: Trace 700 flow chart [34]

The calculations explicitly cover climatic data including hourly temperature, solar radiation, wind and humidity; building orientation, size, shape, mass, and heat transfer coefficients of air and moisture; operational characteristics for occupancy and control modes; mechanical capacities, load performance, ambient wet and dry bulb depression effects; and internal heat gain from lighting, workspaces, and people. All calculations are based on the ASHRAE Handbook of Fundamentals that typically guide industry decision making for engineering design in buildings. Trace 700 was selected for relevancy in the marketplace as well as a mode of comparison with the other methods utilized in this study.

The metric of a typical Midwest home is utilized as a reference throughout the study in addition to Passive House standards. A typical Midwest home is expected to have an energy use intensity of 62 kBtu per square feet (source energy), according to the U.S. Energy Information Administration [11].

3.5.2 Thermal Comfort Analysis

Occupant thermal comfort interviews and field investigations were conducted as part of the initial field investigation as summarized in Table 9. Residents, students and staff who frequent the Gibbs House over the past few years were interviewed for insight into their experiences relating to thermal comfort. The largest finding was based on poor air circulation primarily during the heating season since windows were kept closed. During the cooling season, large oak trees shade south and west sides of the building where solar heat gain would be most prominent. Additionally, the large operable windows allow for natural cross ventilation during the warmer months, which indicates why most complaints are rooted in colder temperature months. Additionally, the building is unoccupied from June through August.

Table 9: Occupant thermal comfort observations

Space	Occupant Thermal Comfort Observations
Living & Dining Rooms	Very cold near windows. Heavy drapes help with blocking cold air and drafts. Radiator casement broken or blocking airflow.
Bedrooms	Poor air circulation. Perimeter heat ineffective at radiating into space.
Bathrooms	Cold near windows. Very little radiant heat provided. Poor ventilation after showers, etc.
Kitchen	Window framing loose which results in draftiness. Little heat provided. Plumbing fixtures leaky and hot water sporadic.

Thermal comfort conditions are considered according to ASHRAE 55-2013 Thermal Environmental Conditions for Human Occupancy [36]. Some windows were observed to be open in January below 30F weather in order to provide fresh air circulation. Occupants noted that fin radiators were prone to not delivering heat on very cold days. This is likely be due to uninsulated piping that does not deliver desired temperatures upon reaching occupant spaces.

The six primary thermal comfort variables are ambient air temperature, radiant temperature, relative humidity, air circulation, metabolic rate, and clothing insulation. These variables are investigated holistically using the University of California Berkeley Center for the Built Environment Thermal Comfort Tool that provides an interface for evaluating the ASHRAE 55-2013 Standard [36] for specific conditions.

3.6 Cost Savings Analysis

RSMeans was utilized for estimating labor hours, number of crew members, base material and equipment costs according to WMU Project Management standard practice. The standard in-house carpenter rate for WMU is \$70 per hour, which was utilized for all labor estimates.

Applying a simple payback time, or return on investment period, as a monetary metric, the alternatives are able to be compared based on both capital cost and energy savings according to Equation 9.

$$Estimated\ Payback\ Time\ (years) = \frac{Estimated\ Capital\ Cost(\$)}{Estimated\ Annual\ Savings\ (\$)} \quad Equation\ 9$$

Although shorter payback times are large drivers for initiating energy savings projects, building occupant comfort is an additional priority. Thus, alternatives are not recommended solely based on payback time. Proposed options are also ranked according to a thermal comfort need as indicated in the previous section.

Chapter 4

4 Results

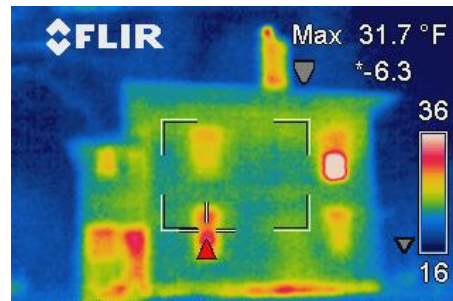
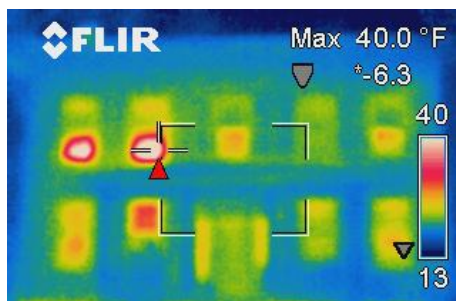
The following section provides results and explanations of the each of the five analysis categories conducted.

4.1 Insulation Analysis

The insulation analysis determined that all main components of the building envelope had greater U-factors than required by code (lower U-factor indicates better insulative value). Passive House standards are more stringent than code [37], [38], thus resulting in an even greater disparity compared to current conditions. As found during the endoscopy investigation, wall insulation was observed to be deteriorated from the original construction in 1863. Thus, the calculated wall U-factor is much greater than designed and currently accepted by code. The 80% increase in U-factor value above code encourages a retrofit to extend the building envelope and provide exterior insulation. Additionally, the current aluminum siding facade is not original. The study also found the windows and doors to have greater U-factors compared to current code. Recommendations include providing wall cavity insulation to all exterior walls. Refer to the Fenestration Results section for window recommendations. Table 10 shows measured temperatures and calculated U-factors during the winter readings (worst case).

Table 10: Calculated U-factors based on IRT

	Avg Surface T (degF)	Code Max U-factor*	Passive House Max U-factor **	Gibbs U-factor	% Increase from Code
Wall	27.125	0.048	0.025	0.243	80.2%
Window	27.75	0.27	0.2	0.453	40.4%
Door	28	0.17	0.17	0.344	50.6%



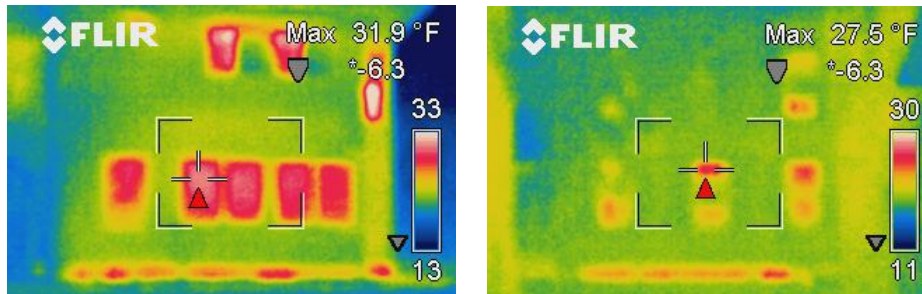


Figure 9: North (top left); east (top right); south (bottom left); west (bottom right) exterior IRT images

Also note the relatively high temperature at the basement ribbon indicating minimal insulation which was verified by visual inspection. The observed heat loss at the ribbon is observable on all building facades. The interior surfaces were also captured using the infrared camera in order to calculate the temperature distribution.

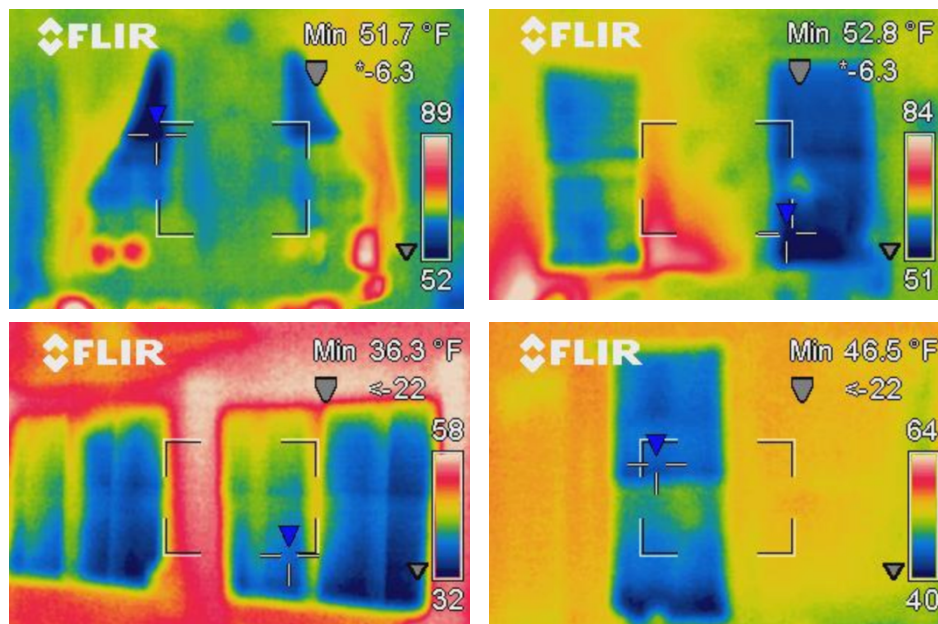


Figure 10: North (top left); east (top right); south (bottom left); west (bottom right) interior IRT images

The visual inspection of the wall cavity found less than 5% insulation volume. The existing insulation appeared to be of the mineral wool type and congregated at the base of the wall cavity, no higher than one foot above the floor level. Additionally, the finned tubed piping picture in Figure 11 is uninsulated resulting in excessive heat loss.

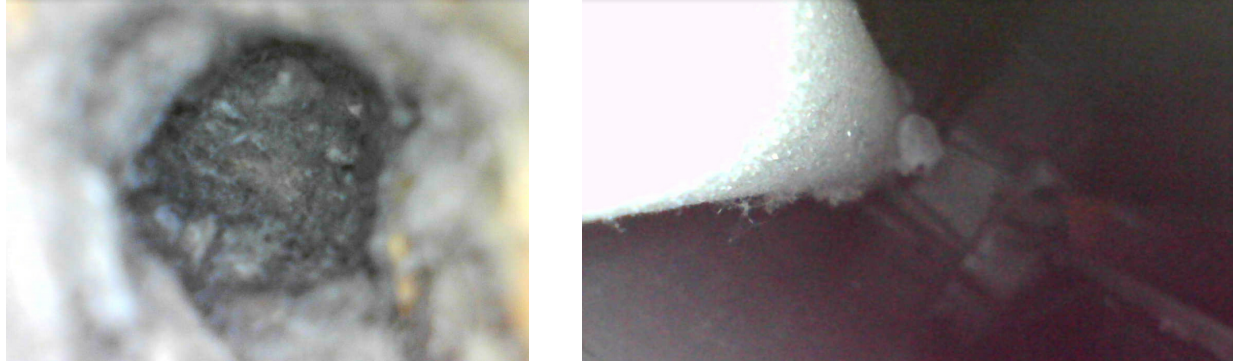


Figure 11: Mineral wool insulation at baseboard (left); empty exterior wall cavity at finned tube level (right)

Insulation values were also calculated based on material properties and as-found interior wall conditions such as air in the mineral wool cavity. Compared to the worst case conditions above, the wall assembly is observed to have a fair total assembly R-value when the air cavity is not subject to extreme temperatures (i.e. worst case observation).

Cellulose is considered an environmentally friendly alternative to fiberglass or mineral wool insulation that also has airtightness qualities. Therefore, it is selected as the blown-in insulation alternative for both interior and exterior additions of insulation. Table 11 provides the thermal performance characteristics as well as the amount of additional thickness that would be required to add on to the envelope in order to provide R-30 cavity insulation, which meets the thermal comfort requirements investigated in a later section. An R-value of R-19 is achievable by blowing in cellulose from the interior for existing framework.

Table 11: Cavity insulation thermal properties

Condition	Thickness (in)	Cavity R-value	k (Btu/hr-ft-F)	Total Assembly R-value
As-Is	3.96	22.15	0.0149 (air)	59.34
Cellulose Blown In, Interior	3.96	30	0.0110	67.19
Cellulose Blown In, Exterior	6.00	45.45	0.0110	83.64

Notes:

Typical R-value of blown in cellulose is 7.6R/in (per rec. manuf.)

Total assembly value is summed since layers occur in series.

Room temperature thermal conductivity values shown.

4.2 Fenestration Thermal Performance Analysis

The fenestration analysis yielded a surprisingly low U-factor for the weatherstripped film alternative at a 52.5% reduction from the as-is window, as compared to the other single pane window options. Additionally, the finite element simulations identified key areas of improvement for the single pane windows such as the air gap located at the sash. As expected, the double pane window yielded the lowest U-factor value to achieve a 76% reduction from the current as-is single pane window.

The combined U-factor calculations provided by the finite element analysis highlighted the effectiveness of weatherstripping and film to resist heat loss during the heating season. As expected, the double pane window was found to have the lowest combined U-factor with the weatherstripped film alternate as second lowest. Repairing, refurbishing, and weatherstripping alone yielded U-factors fairly similar to the existing as-is window with less than 7% reductions. Sample cross sectional geometries are shown in Figure 12.

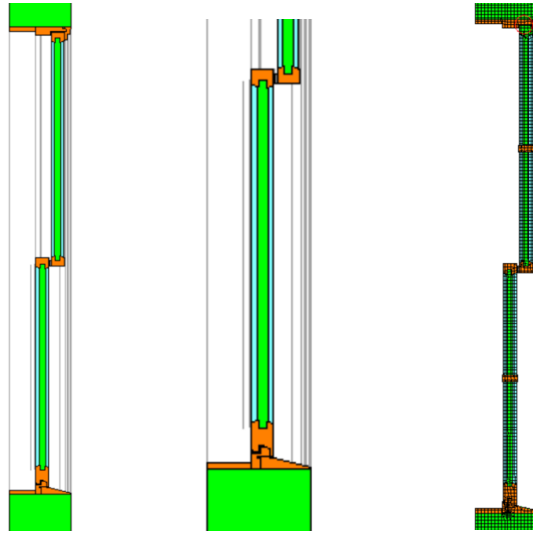


Figure 12: Double pane cross section (left); lower half (middle); full mesh (right) in THERM

The window middle sashes are shown in Figure 13 as a comparison between alternatives. The as-is window has a small air gap that represents the average window gap across all windows: ranging from nearly tight to wide open. The gap is simulated as a highly conductive material (glass) since THERM requires that the boundary conditions are continuous. Additionally, the gap is smaller in width compared to the other single pane alternatives due to this discrepancy so that the glass material assumption would not interfere with the thermal performance of the overall window assembly.

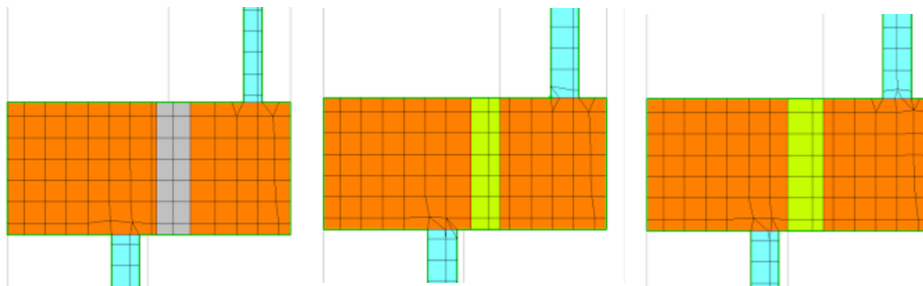


Figure 13: As-is sash (left); repaired sash (middle); weatherstripped sash (right) meshes

The film alternative, however, was able to be computed using THERM's built-in convection algorithm since the area is completely encompassed by the window assembly.

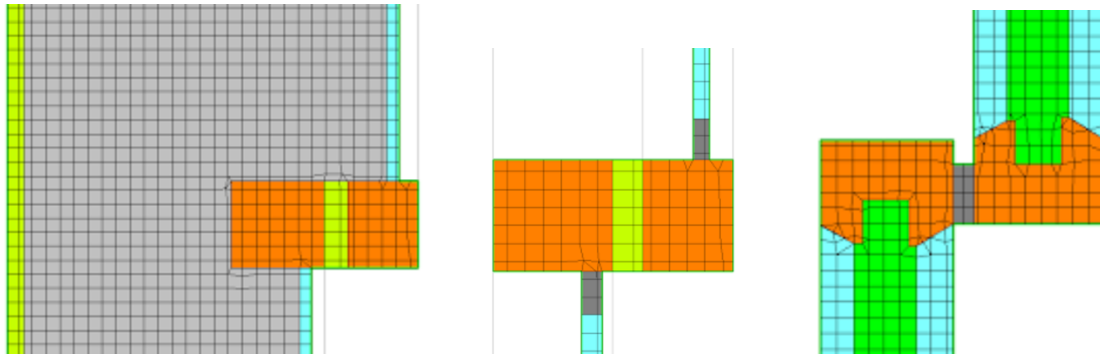


Figure 14: Weatherstripped film sash (left); refurbished sash (middle); double pane sash (right)

Individual material inputs are shown in Table 12 based on IR thermography calculations (Gibbs glass), manufacturer specifications (double pane), and generic software materials (butyl rubber and pine). These material properties were applied to the window section geometries in conjunction with worst case scenario boundary conditions to determine overall U-factors, heat flux and heat flows.

Table 12: THERM boundary conditions

Season	Face	IR Surface T (degF)	Ambient T (degF)	Film Coefficient, h (Btu/h-ft ² -F)
Heating Season	Exterior	26.3	19.3	4.65
	Interior	65.9	62.3	1.84
Cooling Season	Exterior	80.3	85.7	4.65
	Interior	65.9	69.4	1.84

Based on the above inputs, THERM calculated the average U-factor for the entire assembly as shown in the Figure 15. The double pane manufacturer claimed a U-factor of 0.25, whereas the THERM results yielded an overall U-factor of 0.312.

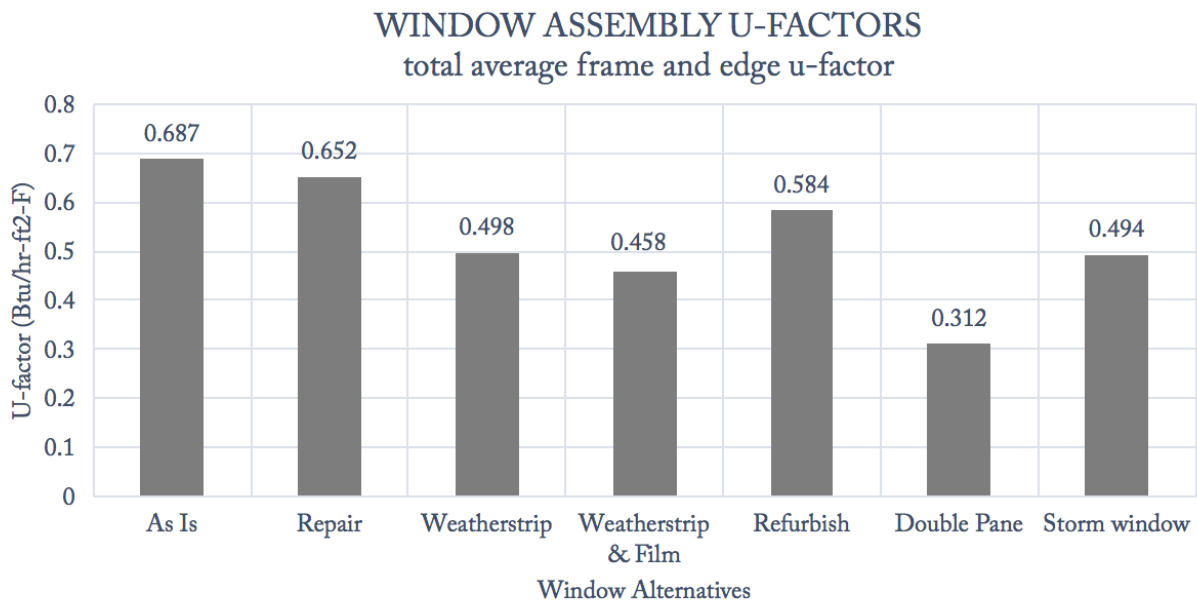


Figure 15: Window assembly U-factors

The middle sash of the double hung existing window was identified as a key area of improvement based on the high heat transfer observed during the finite element simulation. Most windows in the building were observed to have a gap between the front and back window sections. As shown in Figures 16 and 17, the as-is window experiences high heat flux, or in this case heat loss, at the sash. Weatherstripping in the form of EPDM strips was applied in the gap to serve as an insulative barrier. The rendering on the right demonstrates a lesser heat flux (deeper purple hue) as a result of applying the weatherstripping (Alternative B), thus reducing the amount of heat lost to the exterior. The interior of the building is located to the left of each image. A finite element model of a storm window was investigated to reveal similar heat flux patterns to the weatherstripped film option, but with a slightly higher average overall U-factor of 0.494 Btu/hr-ft²-F due to the higher conductivity of glass compared to the film.

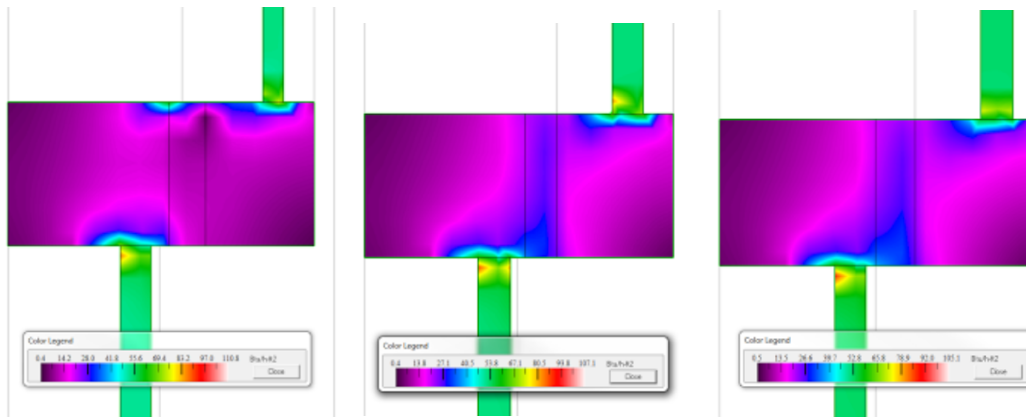


Figure 16: As-is (left); repaired (middle); weatherstripped (right) sash heat flux contours

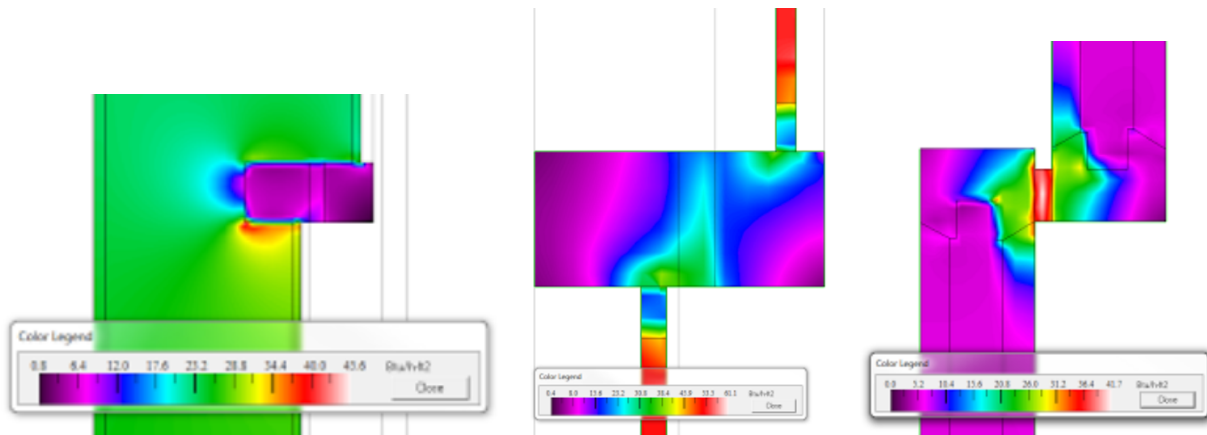


Figure 17: Weatherstripped film (left); refurbished (middle); double pane (right) sash heat flux contours

The temperature contour results for the as-is window (pictured left) demonstrate a dramatic temperature drop from the interior (left side) to the exterior (right side). Additionally, note the lower temperature represented by the blue color on the glass portion of the as-is cross section. Comparatively, the double pane window provides an interior surface temperature around 60 degF across both the frame and glazing edge. This provides increased thermal comfort for the occupants based on reduced temperature swings from thermal radiation.

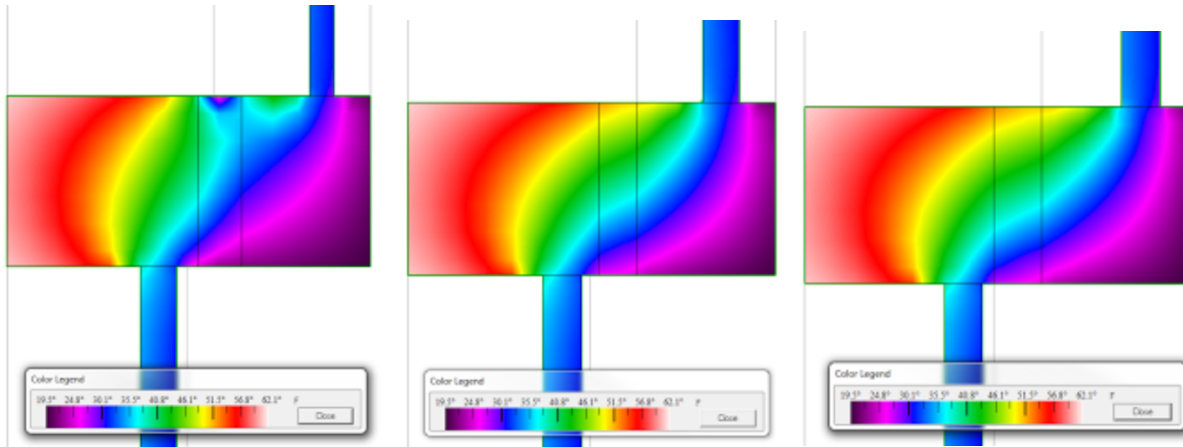


Figure 18: As-is (left); repaired (middle); weatherstripped (right) sash temperature contours

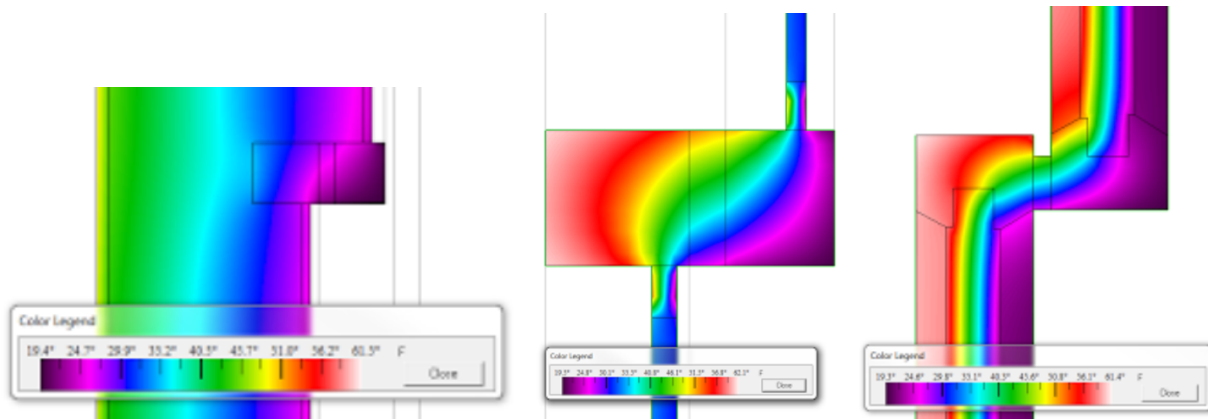


Figure 19: Weatherstripped film (left); refurbished (middle); and double pane sash temperature contours

Weatherstripping and film provides an intermediate solution by increasing occupant thermal comfort compared to the as-is condition. Weatherstripping and film provides an air gap that results in a uniform temperature distribution across the entire window assembly that is encased in film.

On a whole building scale, none of the window alternatives alone were able to achieve Passive House standards with respect to total heating energy and total source energy. Thus, the need for an energy systems upgrade is evident based on the goal of achieving Passive House metrics and replacing the existing out-of-date equipment.

4.3 Building Envelope Analysis

The building envelope was investigated with respect to thermal performance across all wall assembly layers including interior plaster, cavity and rigid insulation, air cavity, and exterior siding. Asbestos and lead testing was conducted by WMU Environmental Health and Safety for the specific areas that may be impacted during renovation recommendations. For any interior modifications made to the baseboards such as adding interior insulation or perimeter radiant heating, it is important to note that lead was found

on baseboard paint. The Environmental Health and Safety representative noted that such areas would not require abatement and renovation methods were possible without added cost, however the future contractor must be made aware of the risks. Abatement costs are likely to be incurred for the removal or refurbishing of exterior windows due to 5% asbestos content found in existing window caulking. Results of this test and all other previous tests can be found in Appendix A.

4.3.1 Two Dimensional Thermal Conduction

Heat conduction through the building envelope is assumed to be steady state, with constant discrete thermal conductivities and no internal heat sources. Therefore, the Laplace equation may be utilized as a special case of the general Fourier heat conduction equation for the building envelope assembly in two dimensions.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad \text{Equation 10 [31]}$$

The building envelope cross section is assumed to be free of internal heat sources or sinks, have constant thermal conductivities within each discrete layer, and have no temperature gradients in the z-direction. The differential equation is solvable by separation of variables if assuming a product solution of the form

$$T = XY \quad \text{Equation 11 [31]}$$

Where

$$X = X(x) \text{ and } Y = Y(y) \quad \text{Equation 12 [31]}$$

The separation constant, γ , is assumed to be λ^2 , in the positive form, in order to consider this boundary value problem as a Sturm-Liouville type characteristic value problem. Thus, the general form of the solution is

$$T(x, y) = (A \sin(\lambda x) + B \cos(\lambda x))(C \sinh(\lambda y) + D \cosh(\lambda y)) \quad \text{Equation 13 [31]}$$

The envelope is comprised of five layers from the interior to the exterior: plaster, mineral wool insulation, rigid insulation, air gap, and aluminum exterior siding (as shown in diagram Figure 20).

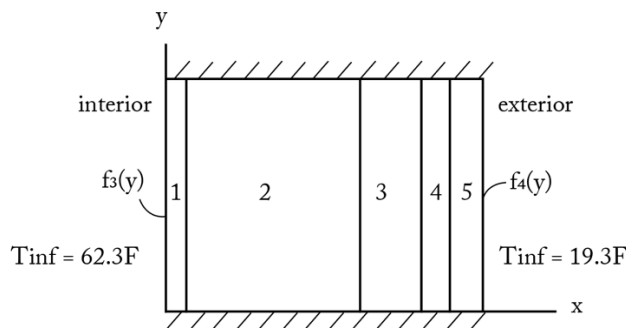


Figure 20: Two dimensional wall assembly diagram

Each layer contributes to the overall temperature distribution via the principle of superposition. The overall temperature profile is defined as

$$T(x, y) = T_1(x, y) + T_2(x, y) + T_3(x, y) + T_4(x, y) + T_5(x, y) \quad \text{Equation 14}$$

Temperature profiles for each layer also utilize the principle of superposition to consider one non-homogenous boundary condition at a time. The problem becomes solvable after applying the homogenous boundary conditions and solving for the constants. For the interior surface of the plaster (left side), the boundary conditions are

$$\begin{aligned} T(0, y) &= f_3(y) & \text{for } 0 < y < b \\ T(a, y) &= f_4(y) & \text{for } 0 < y < b \\ T(x, 0) &= 0 & \text{for } 0 < x < a \\ T(x, b) &= 0 & \text{for } 0 < x < a \end{aligned} \quad \text{Equation 15}$$

Thus

$$T(x, y) = E \sinh(\lambda y) (\sin(\lambda(a - x))) \quad \text{Equation 16}$$

Where

$$E = \frac{-AC}{\cos(\lambda a)} \quad \text{Equation 17}$$

After applying the exterior boundary condition the solution becomes

$$T_e(x, y) = \frac{2}{b} \sum_{n=1}^{\infty} \frac{\sinh\left(\frac{n\pi}{b}\right)x \sin\left(\frac{n\pi}{b}\right)y}{\sinh\left(\frac{n\pi}{b}\right)a} \int_0^b f_3(y') \sin\left(\frac{n\pi}{b}\right)y' dy' \quad \text{Equation 18}$$

Following the same process, and applying the boundary condition, $f_4(y)$, to the interior surface of the wall yields the final solution

$$T_i(x, y) = \frac{2}{b} \sum_{n=1}^{\infty} \frac{\sinh\left(\frac{n\pi}{b}\right)(a-x) \sinh\left(\frac{n\pi}{b}\right)y}{\sinh\left(\frac{n\pi}{b}\right)a} \int_0^b f_4(y') \sin\left(\frac{n\pi}{b}\right)y' dy' \quad \text{Equation 19}$$

Using a linear regression based on infrared thermography plots of the interior and exterior walls, the two temperature functions were determined to be:

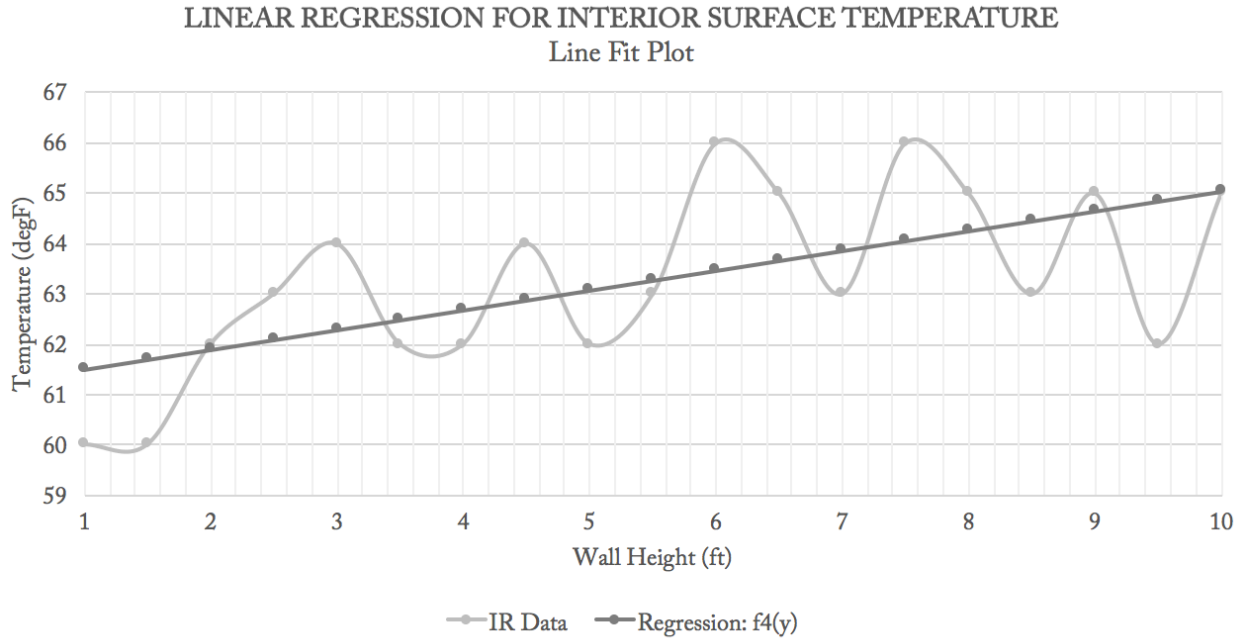


Figure 21: Interior surface linear regression

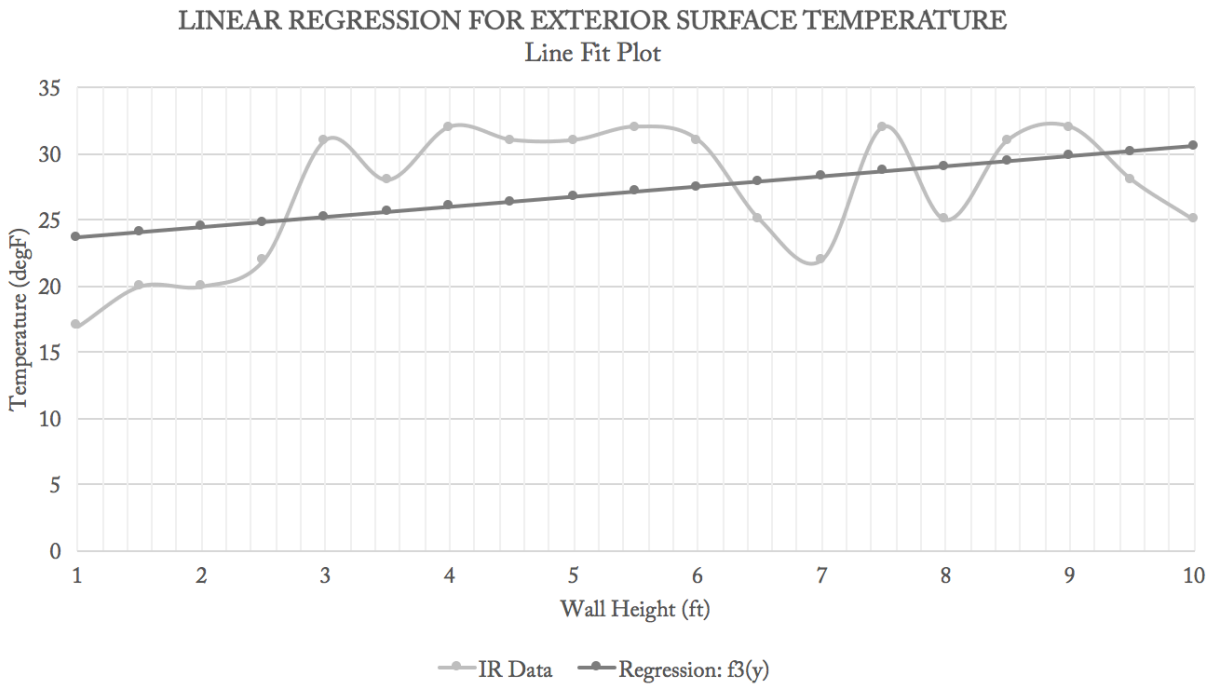


Figure 22: Exterior surface linear regression

The $f_4(y)$ and $f_3(y)$ functions were determined for the exterior and interior surfaces to be

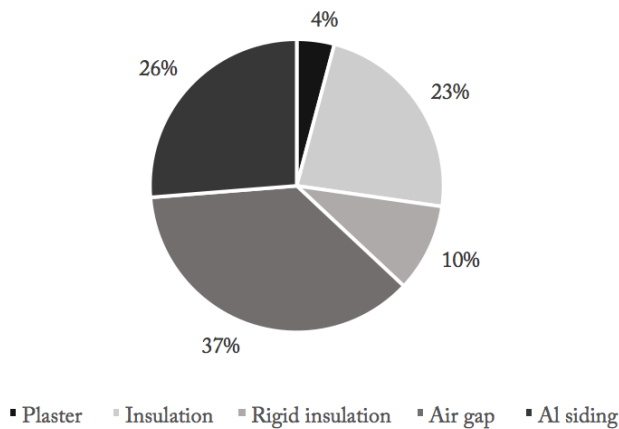
$$f_3(y) = 22.898 + 0.7649y \quad \text{Equation 20}$$

$$f_4(y) = 61.1 + 0.393y \quad \text{Equation 21}$$

Since the y/a values are not extremely small, the Taylor Series is expected to converge rapidly. Thus, the temperature profiles are solved using Mathcad computing software (Appendix B). For uniformity, the thermal behavior is observed at a wall height of five feet to include most building occupants' chest or head height. For simplicity, the air cavity regions are assumed relatively stagnant with negligible buoyancy forces, thus the above equations are solved using a thermal conductivity value of 0.0149 Btu/hr-ft²-F for air. Refer to Equations 22 through 29 for convective considerations within an enclosure.

Figure 23: Manual 2D heat flow results

PERCENTAGE OF HEAT FLOWS BY LAYER
Two Dimensional Manual Calculation



Note from above, the interior and exterior temperature distributions are not identical as expected for a uniform wall assembly without air gaps. However, since the mineral wool cavity insulation was found in a deteriorated state, the effects of natural convection within the wall enclosure are likely more prominent due to this empty cavity. In this case, wall sections 2 (cavity insulation) and 4 (vented air gap) are assumed to be comprised primarily of air. Although the aluminum siding is more thermally conductive than the air gap, the air gap results in greater heat loss due to the greater thickness of the air gap in the direction of heat flow.

Natural convection within the wall cavities occur as a result of conduction boundary conditions on both vertical walls. The fluid is assumed to be Newtonian and incompressible with no internal heat generation present. Initially, at time zero, the respective vertical wall surfaces are subject to different temperature distributions within the envelope.

The two dimensional governing equation is based on the mass conservation equation for an incompressible fluid:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{Equation 22 [39]}$$

This configuration is an exact differential and is set equal to $\delta\psi$ and becomes

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad \text{Equation 23 [39]}$$

Specifically, natural convection within a stationary reference frame can be expressed as

$$\rho \frac{DV}{Dt} = \rho g - \nabla p + \nabla \cdot \tau \quad \text{Equation 24 [39]}$$

Natural convection follows the Boussinesq assumption where the continuity equation is

$$\nabla \cdot V = 0 \quad \text{Equation 25 [39]}$$

Thus the momentum equation follows as

$$\rho \frac{DV}{Dt} = \rho g - \nabla p + \nabla \cdot (\mu \nabla V) \quad \text{Equation 26 [39]}$$

The governing equations are based on a combination of the mass conservation equation and the viscosity relationship

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad \text{Equation 27 [40]}$$

Applying a proportional analysis on the energy equation above by assuming x is represented by δ_t and y is represented by H , the method of scales proposed by Faghri [63] for the second order portion of the equations yields

$$\frac{\Delta T}{\delta_t^2} \text{ and } \frac{\Delta T}{\delta_t^2} \quad \text{Equation 28 [40]}$$

Since the height of the wall is on the scale of approximately 10 feet and the depth of the wall is less than 1 foot, one can state $\delta_t \ll H$ and conclude that

$$\frac{\partial^2 T}{\partial y^2} \ll \frac{\partial^2 T}{\partial x^2} \quad \text{Equation 29 [40]}$$

As shown by the relationship above, the temperature gradient in the x -direction is much more prominent than in the y -direction. The resulting convective air flow pattern is clockwise circulation and is deemed to have insignificant effects on overall heat transfer in the wall. Thus, the simplified conduction calculations stated early are accepted as adequate to represent the overall thermal performance of the building envelope.

4.4 Airtightness Analysis

Window sashes were determined as small driving factors for air leakage during the blower door testing. However, physical observation highlighted noticeable air leakage at the sash when standing nearby. On a whole building level, implementing weatherstripping at the sash and sides provided a 3% drop in airflow compared to the existing case. For the standalone window tests, implementation of weatherstripping and film resulted in a 2.1% reduction from the existing room leakage.

4.4.1 Whole Building Depressurization Testing

A blower door study was conducted in 2007 by Alex Rensch, a Western Michigan University student, and a local blower door contractor, Robert Kildea. The team determined the air leakage and HERS rating score, as shown in Table 13. The study focused on basement improvement measures based on an observed lack of insulation, especially around the ground level ribbon.

Table 13: Depressurization test results from 2007

Metric	Initial State	Final State	Result
CFM @ 50Pa	3400 CFM leakage	3100 CFM leakage	300 CFM decrease after adding basement insulation
HERS Score	145	140	5% increase

Preliminary blower door studies conducted for this research occurred in August 2017. Initially, measurements were taken with no modifications to the building other than mandatory building preparations such as covering exhaust ducts and shutting down natural gas equipment. Refer to the photos in Figure 24 for examples of envelope deficiencies. As indicated by Table 14, the record air leakage at 50Pa is agreeable with the final value recorded in 2007. Surprisingly, the building envelope appears to have experienced minimal infiltration depreciation over the last ten years.



Figure 24: Solarium window gap (left); pane glass crack and deteriorating caulking (middle); pane crack (left)

Temporary modifications included securing loose and unstable window assemblies with weatherstripping tape in order to simulate a properly installed window. The preliminary results represent possible air tightness associated with single pane historic windows that have been restored, repaired, and reinstalled correctly to avoid unnecessary gaps and cracks as well as other non-fenestration infiltration issues.

The average leakage rates and observed leakage locations are shown in Table 14. Leakage behavior over the sample periods are shown in Figure 25 for each test case.

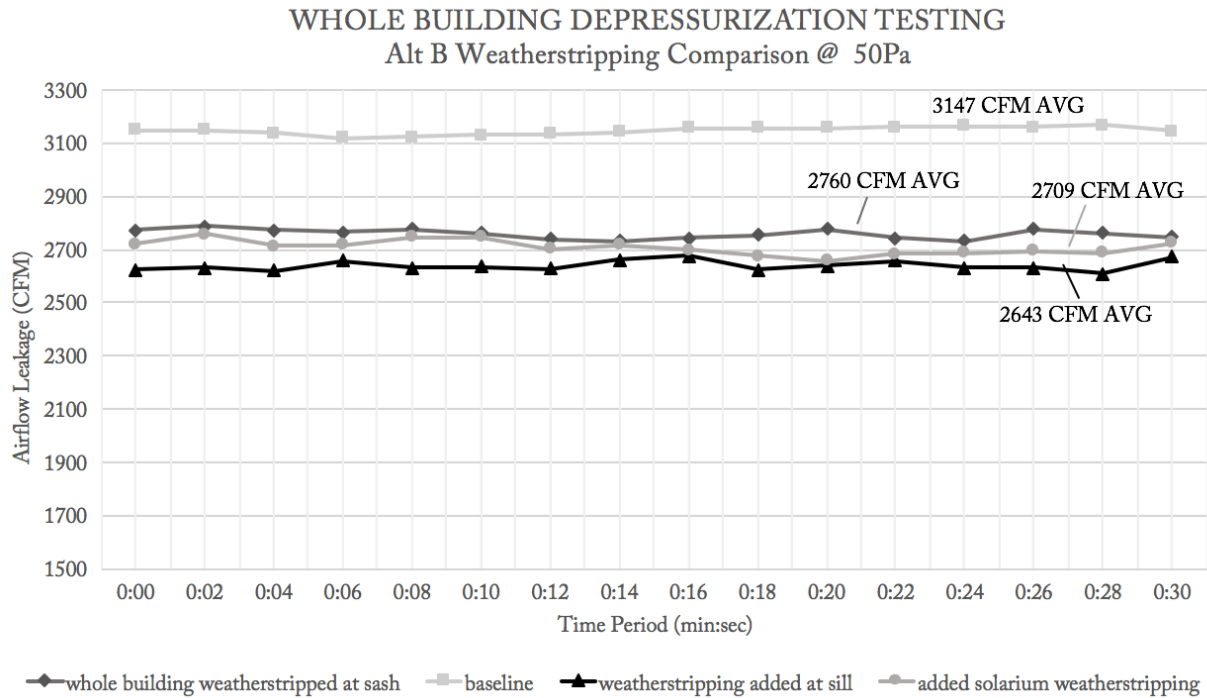


Figure 25: Whole building depressurization test for weatherstripping

Whole building depressurization testing results are tabulated in Table 14, including air changes per hour. The Passive House standard for airtightness is 0.6 ACH. Even after weatherstripping all the sashes and sills the air change rate is 75% greater than the Passive House recommendation. This large discrepancy indicates that the building is nonhomogeneous and leakage occurs at other locations in the building in addition to the windows. For instance, additional leakage points are likely to occur in the basement and attic as extensively air sealing has not been implemented in these areas.

Table 14: Whole building average depressurization test results summary

Test	Average Air Leakage (CFM)	Air Changes per Hour (ACH50)	Physical Observations
Existing 2007 results	3100	2.79	Leakage around outer window framing
As-Is baseline	3147	2.84	Leakage at outer frame, mid-sash, and basement ribbon
Weatherstripped at sash	2760	2.49	Leakage at frame sides
Weatherstripped at sash and sill	2643	2.38	Noticable drop in airflow leakage

4.4.2 Standalone Window Depressurization Testing

Visual representations of the blower door results are shown in Figure 26 over a one minute sample period. Per the Passive House standard, measurements are taken at a building pressure of 50Pa as a control. The standalone study for weatherstripping involved installing a control window in the frame of a one window bathroom at the Gibbs House. First, testing was conducted without the window installed (room baseline). Then, the window was installed with EPDM weatherstripping at the sash. The next round of testing included adding weatherstripping to the side gaps of the double hung window. There was very little drop in airflow with just the weatherstripping installed at the sash. However, the combination sash and sidewall weatherstripping provided a small drop in airflow (3%).

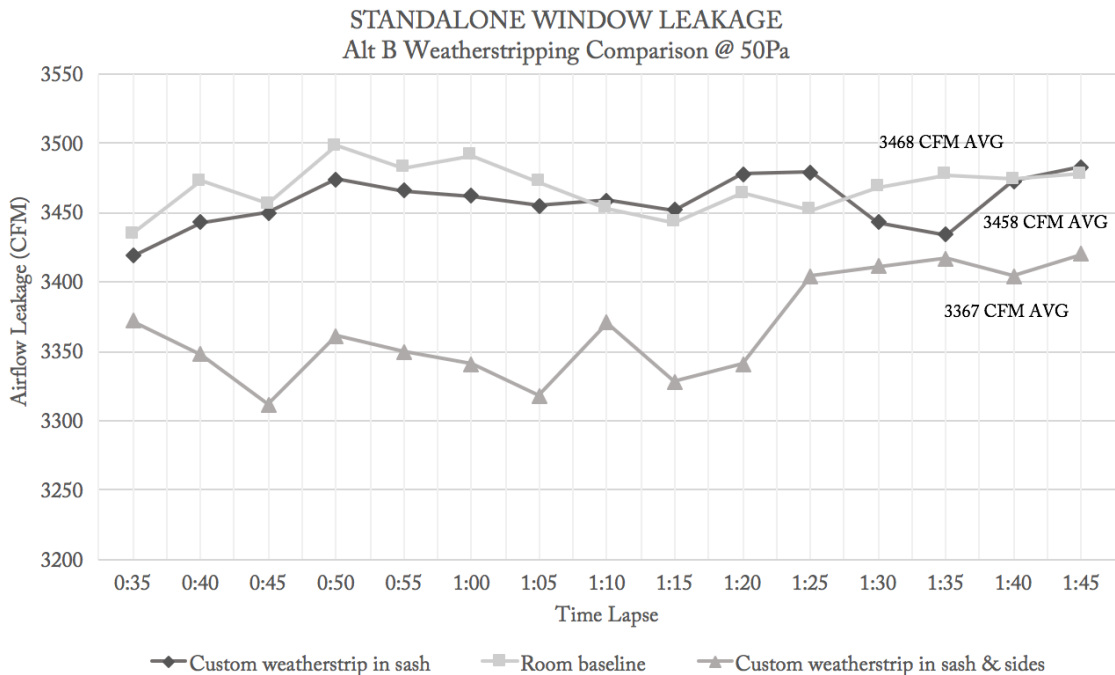


Figure 26: Standalone window comparison for weatherstripping

The same process was employed for refurbishing a window. The refurbishing process included deconstructing the window by removing old caulk and then recaulking to secure the existing glass panes. The window was then installed under the same conditions and compared against the room baseline. The retrofitted window provided no decrease in airflow compared to the baseline, likely due to the poor original frame construction that is difficult and timely to repair. Improving this aspect of the window would essentially result in building a new window. However, a small drop in airflow (2%) was observed when weatherstripping and film was provided for the window.

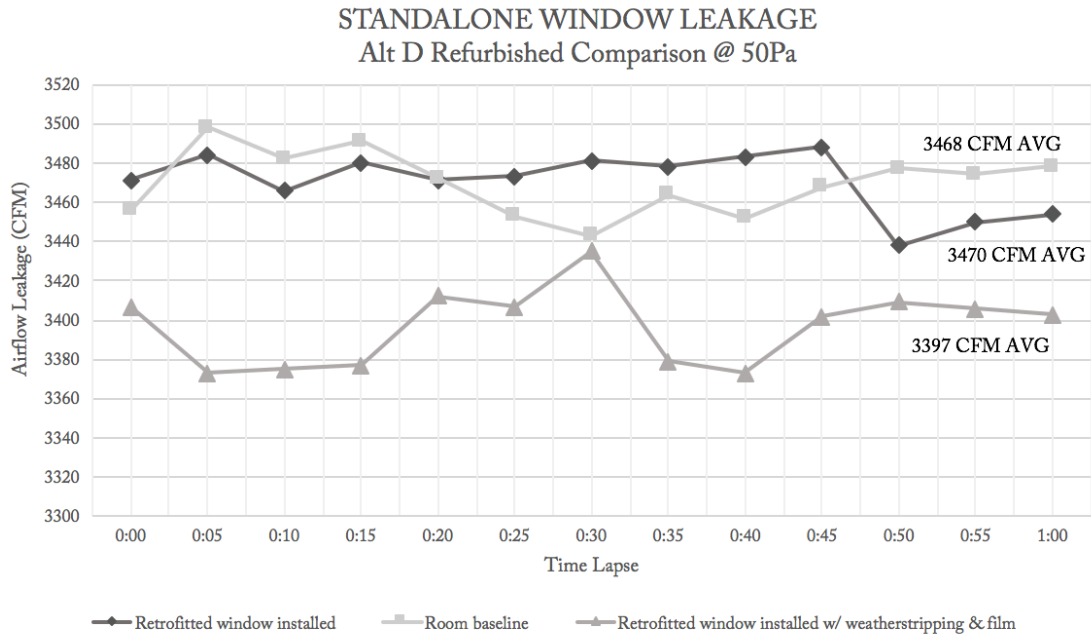


Figure 27: Standalone window comparison for refurbishing

In general, the tedious and time consuming work required to weatherstrip, install film, and refurbish old windows may not be worth the overall building infiltration benefits compared to air sealing in other locations such as the attic and basement. The testing revealed small reductions in airflow when weatherstripping and film were installed, which may provide occupants with an increased sense of thermal comfort that alleviates common drafts and discomforts.

4.5 Energy Systems Analysis

Autodesk Revit provided Building Information Modeling tools such as 2D and 3D renderings, equipment sizing, and exported building data for energy modeling in Trace 700.

4.5.1 Mechanical, Electrical, and Plumbing Systems Analysis

The whole building energy model was conducted in Trane Trace 700 for each of the proposed alternatives. In general, the energy model follows the actual building patterns for energy consumption. For instance, gas consumption increases in the winter months due to increased use of the natural gas boiler to provide heating hot water to baseboard radiant heaters. Figures 28 and 29 demonstrate the synchronicities as justification for utilizing the modeling program to predict future energy behavior on an annual basis.

Actual energy consumed at the Gibbs House is routinely transient and unpredictable. Energy use is primarily dependent upon resident schedules, volunteer site visits, hosted events, outdoor conditions that may move activities indoors, and weather that drives heating needs. Energy is considered on an annual basis for the purposes of comparing energy cost savings and payback times to account for inconsistencies across months. Trace 700, the energy modeling software, sums monthly energy use to calculate annual energy use. Therefore, monthly discrepancies between actual and modeled energy are explored to understand energy use patterns that deviate from those expected.

On an annual basis, the modeled electricity uncertainty compared to actual gas consumption from the previous year is 3.13%. Since estimated cost savings and simple payback times are analyzed in this report on an annual basis, the uncertainty is assumed to be agreeable for comparative purposes. The model runs according to a typical residential schedule expected by the residents: occupied on weekends and on weekdays after 5 PM. Occupancy levels were modeled at a lower value for the months of June, July and August to reflect no permanent residents and only permaculture team members and site volunteers. Actual energy patterns experience peak values from February through May whereas the model reveals a relatively consistent energy use across all months. This is likely a result of unforeseen fluctuations in energy use due to events and potlucks, staff and student meetings, and impromptu office space for staff due to weather during normal business hours where the building would otherwise be unoccupied. Since the only building equipment that relies on electricity is lighting, exhaust fans, appliances, pumps and other plug loads, the discrepancies can be associated with the occupancy swings, as discussed.

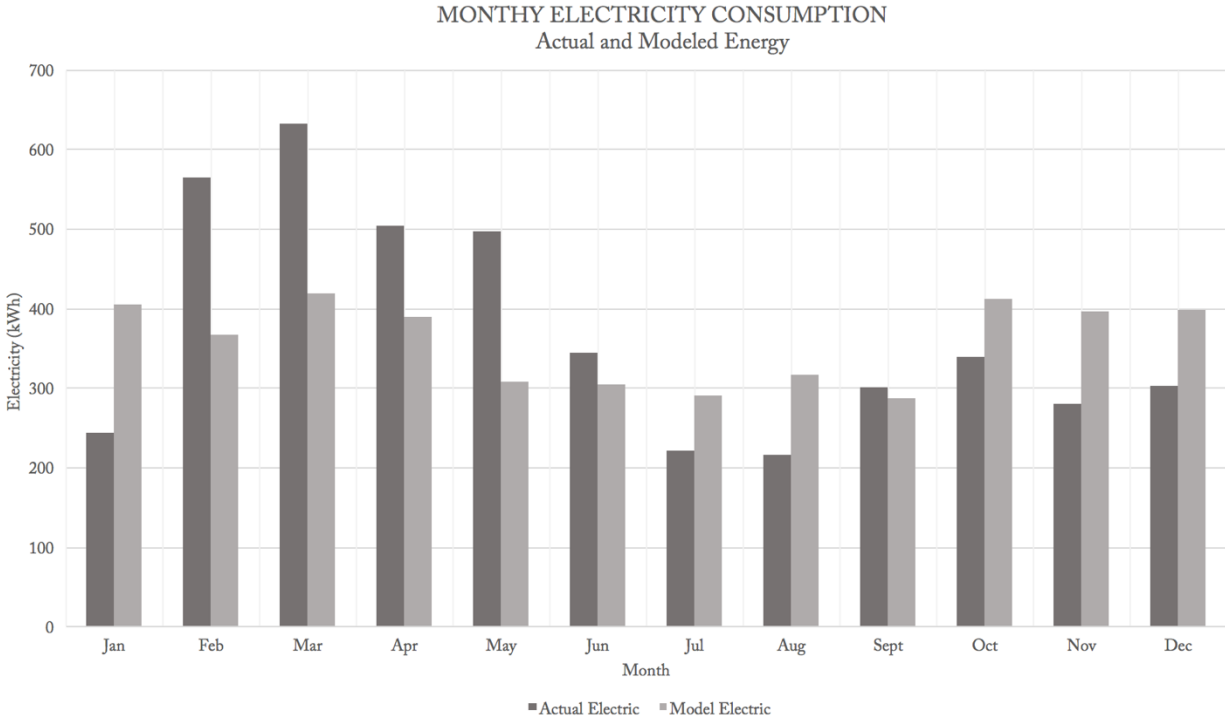


Figure 28: Actual and modeled monthly electricity consumption

Modeled gas consumption uncertainty compared to actual gas consumption from the previous year is 4.5% on an annual basis. Similar to electricity use, the model reflects a more consistent energy use pattern across all months. Whereas actual gas use reflects very large swings across summer and winter months. During the two coldest months, December and January, heating equipment is operating at maximum capacity, as reflected by the very high gas usage and failing to meet occupant needs through ineffective perimeter heat. Additionally, domestic hot water is provided by the natural gas water heater which experiences demand peaks during these colder months. Major drops in energy usage are observed during the summer months are also observed due to no cooling equipment being present in the building. Since the boiler and water heater are the only equipment that rely on natural gas in the building, there is virtually no demand when residents are not present to initiate a call for heat. The model is conservative in projecting gas usage during these warmer months due to visitors.

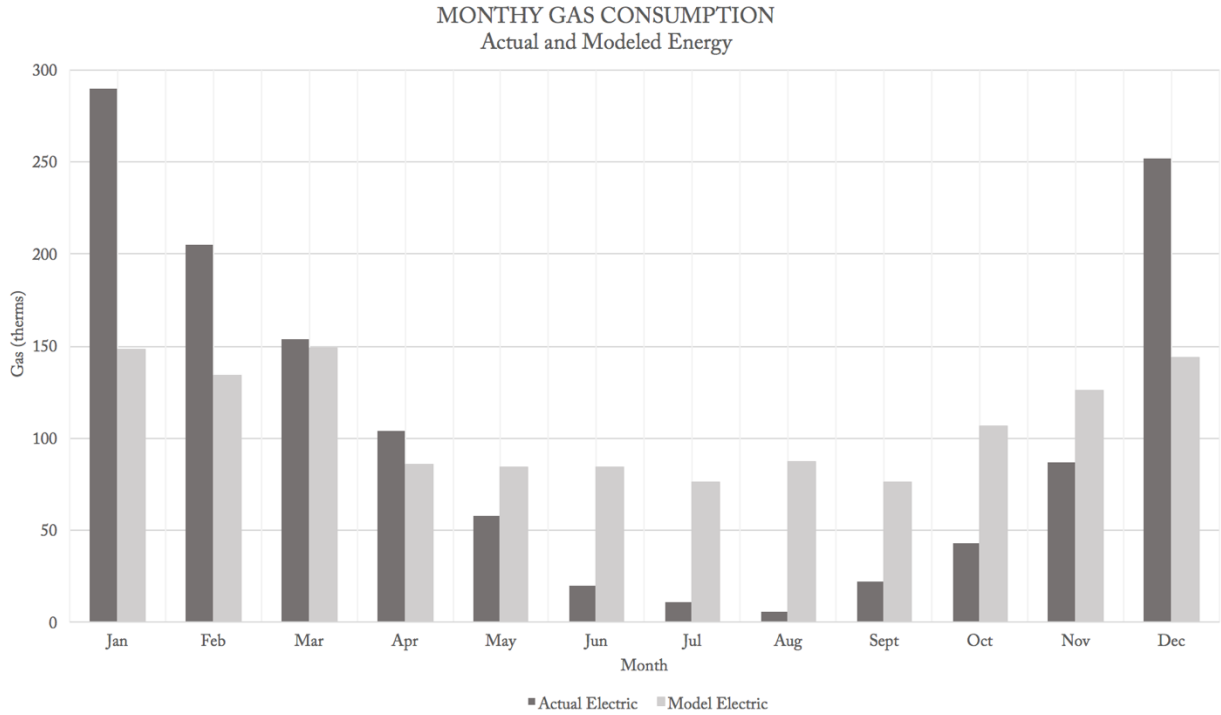


Figure 29: Actual and modeled monthly gas consumption

Every facility improvement measure was modeled in Trace 700, including the as-is condition (Alternate 1). Alternatives denoted by the number two include a mechanical upgrade, while alternatives denoted by the number three are based on only window modifications. Letters indicate a type of window alternative as explained in detail in the Fenestration Thermal Performance Analysis section. Alternative 4 represents a fully retrofitted building envelope that involves removing the existing siding, re-insulating from the exterior and installing new double pane windows. No energy system upgrades are considered. Alternative 5 represents a deep retrofit including energy system upgrades, new building envelope, and new double pane windows. An additional option is added onto Alternative 2 to include mechanical ventilation via a ductless mini-split air conditioning unit. This option would be able to provide heating and cooling and filtered air circulation to the living room. However, installation would require penetration of the wall assembly, which could result in possible thermal bridging and impede the historical aesthetic of the space.

Per Passive House standards, energy performance is judged based on two main metrics: total primary heating energy for equipment and total source energy provided to the building via utilities or on-site generation. As expected, the two deep retrofits (Alternatives 4 and 5) achieve the lowest energy in both categories due to energy efficient equipment, high performing windows, and increased insulation at the building envelope. Alternatives 2C, 2E, 3C, and 3E are also interesting, however. Alternatives with letter C are based on weatherstripping and filming existing windows and yield relatively low energy totals compared to the as-is case and other lettered alternatives. The E alternatives include new double pane windows and result in less total consumed energy based on their greater insulative value.

MODELED ENERGY CONSUMPTION All Base Alternatives Comparison

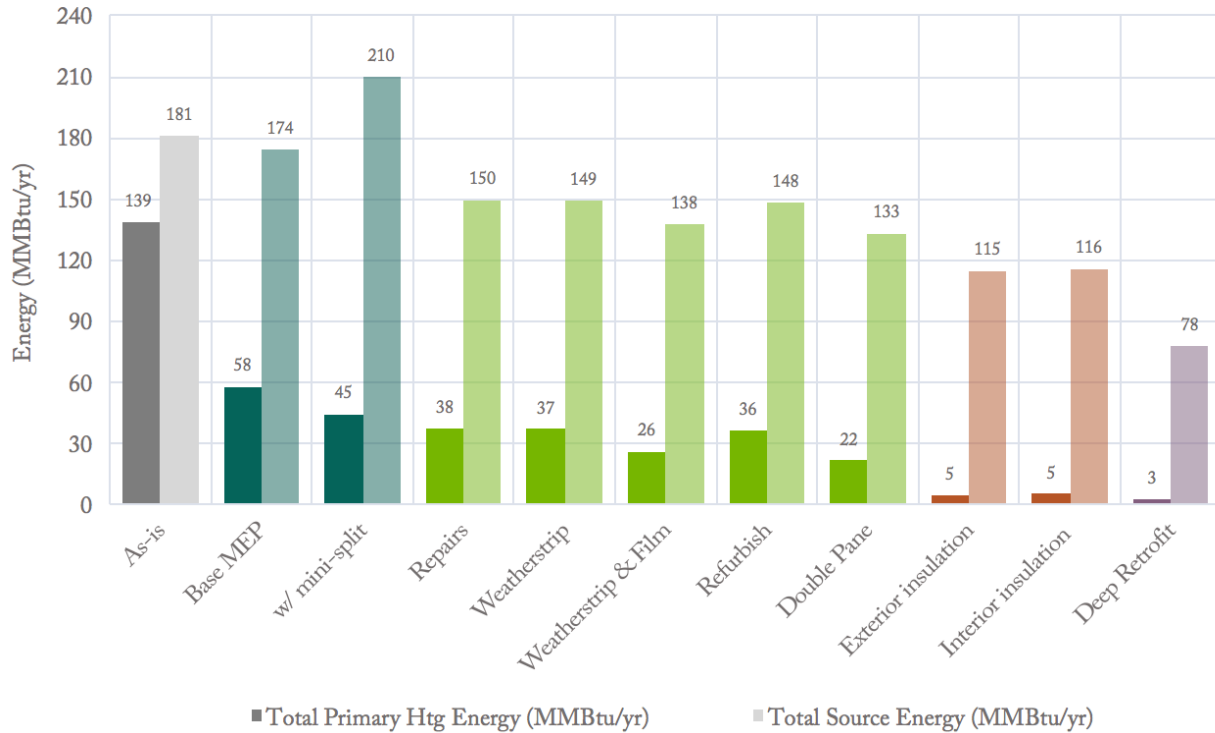


Figure 30: Modeled energy consumption by alternative

For ease of focusing on one concept at a time, only the energy systems upgrade (Alternative 2) options are considered in Figure 31. All options but the mini-split add-on result in lower source energy compared to the as-is model. This is due to the increasing of mechanical equipment which drives the increase in electrical load, since the unit is run on electricity. Therefore, the comparison is difficult to make since the house currently does not provide mechanical cooling. However, by upgrading the boiler and efficiency of radiant heat, the primary heating energy drops to far below half of the current heating energy required.

MODELED ENERGY CONSUMPTION MEP Upgrade Alternatives Comparison

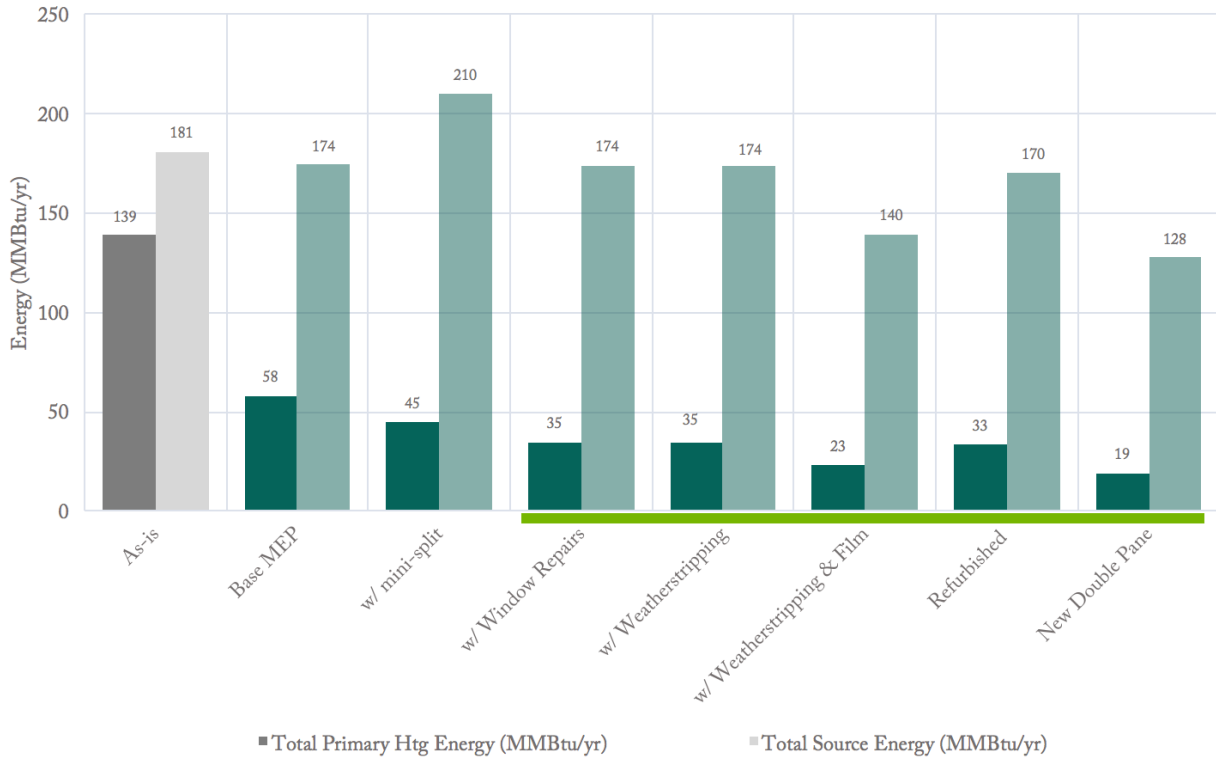


Figure 31: Modeled energy consumption for MEP upgrade options

Focusing on whole building source energy highlights the increase on energy consumption if mechanical ventilation and cooling is provided to the building, as shown by the negative value in Figure 32 for the mini-split option. Since the building has no precedent for mechanical ventilation, any additions of equipment will increase building source energy despite any energy efficiency claims.

SOURCE ENERGY SAVINGS Compared to Modeled As-Is Energy Source Consumption

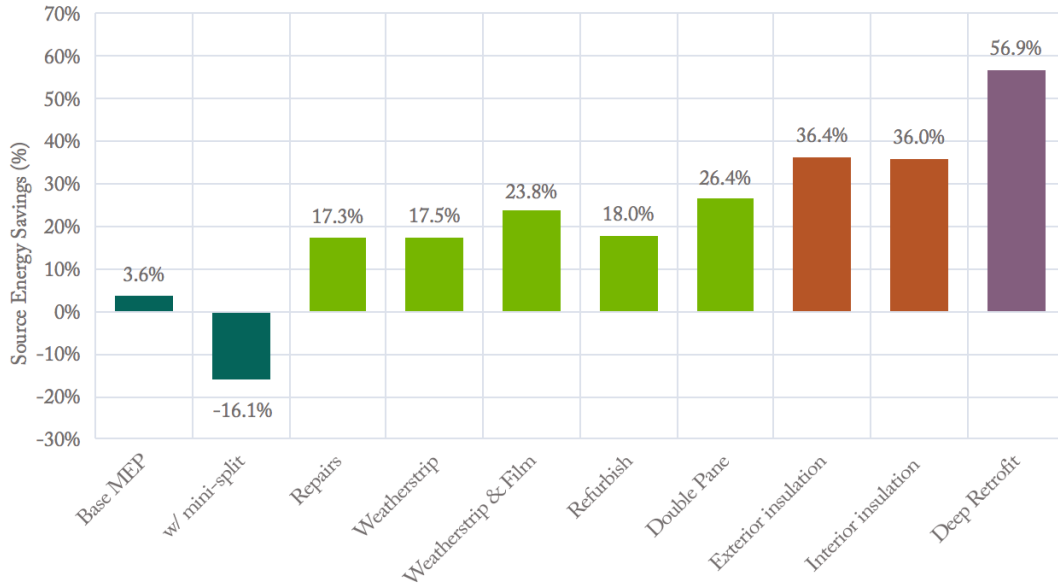


Figure 32: Source energy savings percentages

The Passive House requirement for total heating energy is shown in Figure 33 as the black horizontal line. Alternatives 4 and 5 are the only options that achieve the target according to the simulated Trace model. This identifies that Passive House standards are not achievable with window upgrades alone and high performance windows are likely required as well.

TOTAL ANNUAL HEATING ENERGY PER SQUARE FOOT Compared to Passive House Standard

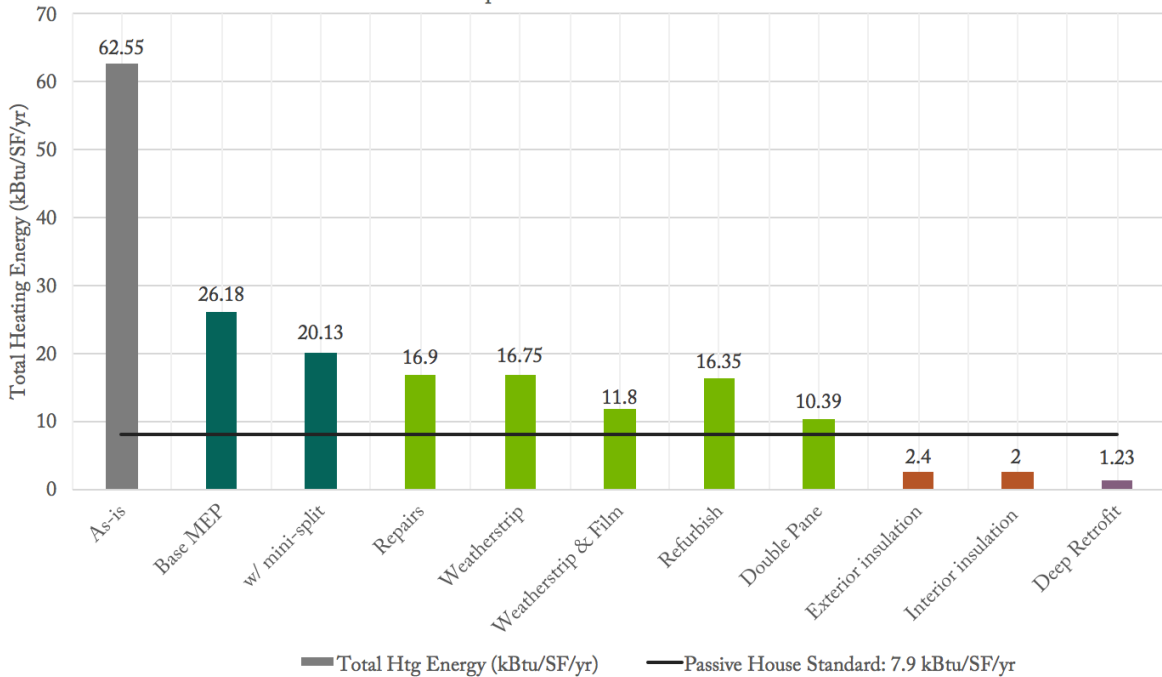


Figure 33: Total annual heating energy compared to Passive House standard

A similar phenomenon occurs for total annual source energy: Alternative 5 is the only option that achieves the Passive House metric. Alternative 5 includes exterior insulation and new siding, new mechanical upgrades, and new double pane windows.

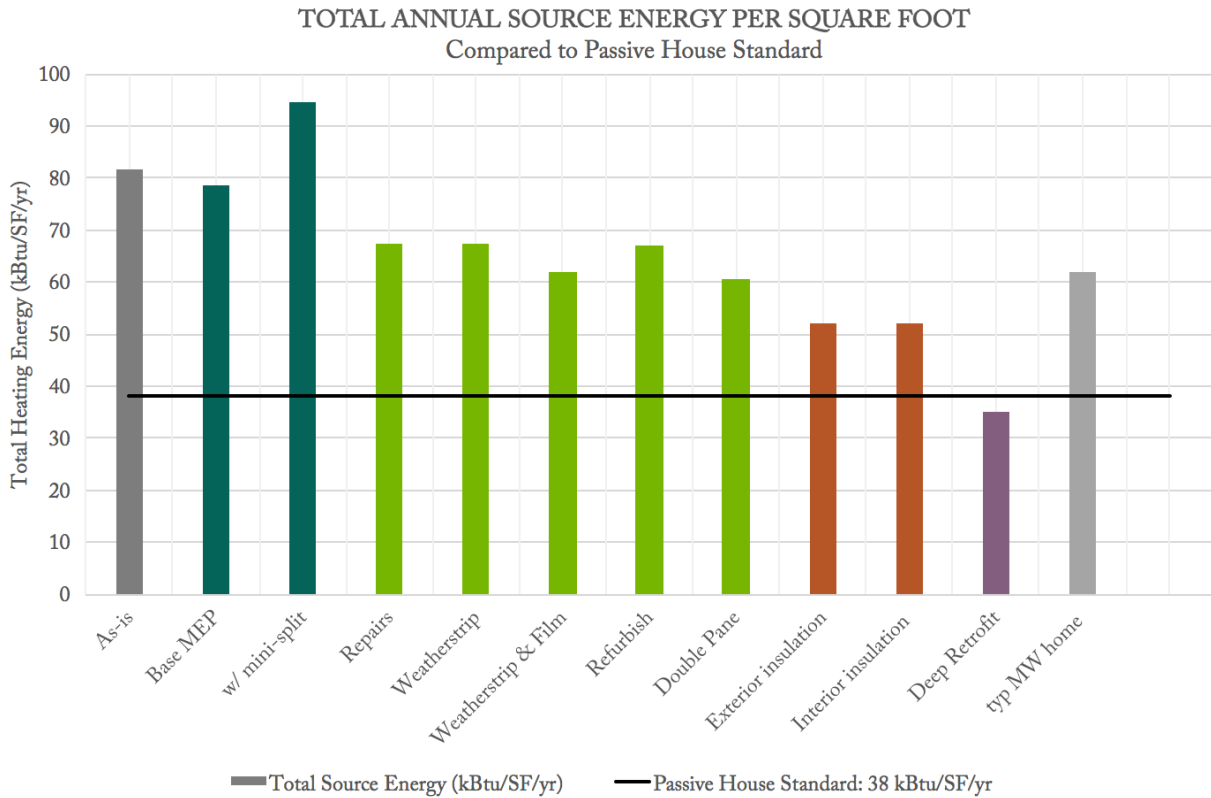


Figure 34: Total annual source energy compared to Passive House standard

4.5.2 Thermal Comfort Analysis

The CBE Thermal Comfort Tool provides an informational interface to determine compliance with the ASHRAE Standard 55-2013. As shown by the psychrometric chart in Figure 35, the Gibbs House does not meet compliance with the Standard due to cold thermal radiation sensations, and a dry bulb temperature (red dot) outside of the target region (purple shading). The instance shown is for the heating season where the air temperature is 62.6 degF, the mean radiant surface temperature is 66 degF, and the average relative humidity is 35%RH. The sensation determined by the comfort tool was cool.

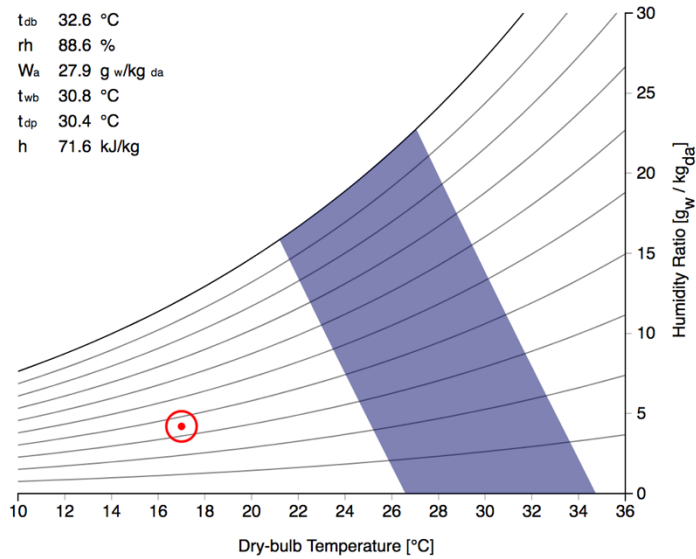


Figure 35: As-is thermal comfort results

The poor thermal comfort observations indicate a need for implementing mechanical ventilation that provides fresh air and regulated humidity control such as through an energy recovery unit.

After implementing exterior wall insulation and installing an energy recovery unit, the building is likely to meet ASHRAE 55 compliance as shown in Figure 36. The WMU standard for occupancy temperature setpoints are 71F and 74F for heating and cooling, respectively. Thus, these setpoints are recommended for initial control sequences for the building. An air temperature of 74 degF was utilized to meet the standard setpoints. Since the building use is residential, these setpoints are adjustable. The minimum mean radiant temperature, or surface temperature, is required to be 68F in order to meet ASHRAE 55 compliance. Since the existing R-value of the cavity insulation is R-22, the insulation would have to increase to be at least R-30, as calculated by the two dimensional building envelope program developed for this study in an earlier section. Relative humidity levels are recommended to be controlled to less than 65% to reduce the likelihood of conditions that can lead to microbial growth [41]. The same metabolic rate of minimal seated work, 1.0 met, and typical winter wear, 1 clo (1.136 Btu/hr-ft²-F), were used as a control. The sensation determined by the Comfort Tool was neutral.

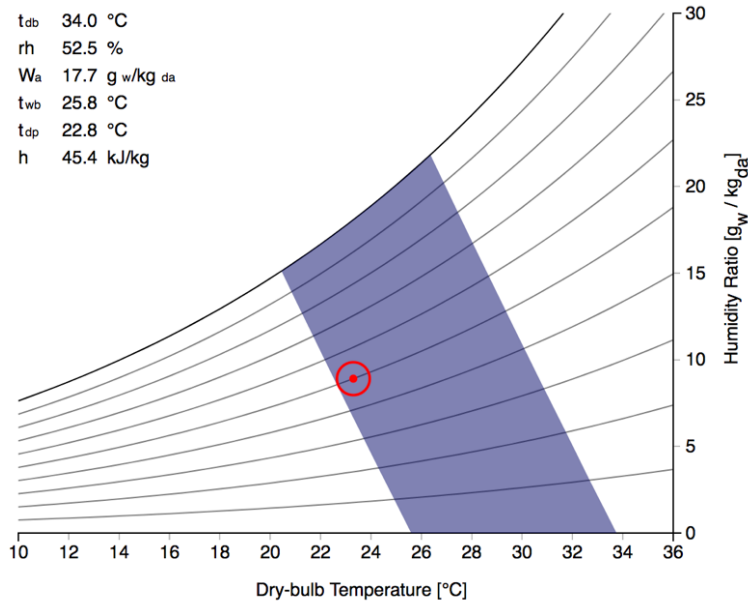


Figure 36: Minimal threshold thermal comfort results

4.6 Cost Savings Analysis

The cost estimates included in this analysis are based on general material, equipment and labor costs according to RSMeans for Kalamazoo, Michigan. The cost estimate method used was recommended by WMU Campus Planning as a preliminary assessment, but more accurate costs are not determined until contractor bidding occurs. Therefore, the estimates provided serve as an introductory tool to assess the range of possible alternatives as well as investigate impacts on associated simple payback times. The breakdown of each of the alternative estimated costs are provided in Tables 15, 16 and 17 based on RSMeans, Kalamazoo prevailing wages, and the WMU labor rate of \$70 per hour for a standard contractor. The cost estimates do not include specific energy metering and additional controls for future research methods since these costs relate to the final construction method selected.

Sporadic traces of lead and asbestos were detected, thus cost abatement premiums depending on the selected alternatives must be determined by the contractors. WMU Environmental Health and Safety is able to test in specific locations prior to hiring contractors.

Additional mechanical options are considered for ventilation. The mini-split air conditioning unit and packaged electric unit do not require ductwork, which is able to preserve more of the historic aesthetic and wall construction. However, due to occupant thermal comfort concerns, an energy recovery ventilator (ERV) is included in Table 15. The ERV requires ductwork, and additional information regarding this upgrade is available in the Energy Systems Results section.

Table 15: Mechanical, electrical, and plumbing cost estimate

	Unit Material Cost	Quantity (ea.)	Total Material Cost	Labor Hours	Local Labor Cost	Local Total Cost	WMU Labor Cost	Total WMU Cost
Equipment demolition	-	-	-	40	\$ 2,000.00	\$ 2,000.00	\$ 2,800	\$ 2,800
Replace boiler	\$ 5,500	1	\$ 5,500	20	\$ 1,000.00	\$ 6,500.00	\$ 1,400	\$ 6,900
Replace water heater	\$ 1,200	1	\$ 1,200	20	\$ 1,000.00	\$ 2,200.00	\$ 1,400	\$ 2,600
Replace finned tube	\$ 1,000	1	\$ 1,000	30	\$ 1,500.00	\$ 2,500.00	\$ 2,100	\$ 3,100
New attic fan	\$ 1,000	1	\$ 1,000	10	\$ 500.00	\$ 1,500.00	\$ 700	\$ 1,700
New ceiling fans	\$ 1,000	5	\$ 5,000	10	\$ 500.00	\$ 5,500.00	\$ 700	\$ 5,700
LED lighting	\$ 100	5	\$ 500	10	\$ 500.00	\$ 1,000.00	\$ 700	\$ 1,200
Piping insulation	\$ 100	1	\$ 100	10	\$ 500.00	\$ 600.00	\$ 700	\$ 800
<i>Options:</i>								
Mini-split AC unit	\$ 1,300	1	\$ 1,300	10	500	\$ 1,800	\$ 700	\$ 2,000
Packaged electric cabinet A/C unit	\$ 1,000	1	\$ 1,000	20	1000	\$ 2,000	\$ 1,400	\$ 2,400
Energy recovery ventilator	\$ 1,000	1	\$ 1,000	20	1000	\$ 2,000	\$ 1,400	\$ 2,400
Ductwork, demo, etc.	\$ 800	1	\$ 800	30	1500	\$ 2,300	\$ 2,100	\$ 2,900
Total Cost, Base option			\$ 14,300	150	\$ 7,500	\$ 21,800	\$ 10,500	\$ 24,800
Total Cost, Mini-split option			\$ 15,600	160	\$ 8,000	\$ 23,600	\$ 11,200	\$ 26,800
Total Cost, ERV option			\$ 16,100	200	\$ 10,000	\$ 26,100	\$ 14,000	\$ 30,100

Alternative 3 is based on window alternatives. As expected, the double pane window results in the highest cost, relatively. Refurbishing of the 30 windows is also costly and time consuming, as demonstrated in Table 16. The premium for in-house carpentry work is shown as well to provide transparency in evaluating recommendations.

Table 16: Window cost estimate

Name	Unit Material Cost	Quantity (ea.)	Total Material Cost	Labor Hours	Local Labor Cost	Local Total Cost	WMU Labor Cost	WMU Total Cost
Repairs	\$ 100	30	\$ 3,000	100	\$ 1,500	\$ 4,500	\$ 7,000	\$ 10,000
Weatherstrip	\$ 10	30	\$ 300	40	\$ 600	\$ 900	\$ 2,800	\$ 3,100
Weatherstrip & film	\$ 20	30	\$ 600	60	\$ 900	\$ 1,500	\$ 4,200	\$ 4,800
Refurbish	\$ 300	30	\$ 9,000	400	\$ 3,200	\$ 12,200	\$ 28,000	\$ 37,000
New double pane	\$ 650	30	\$ 19,500	300	\$ 8,100	\$ 27,600	\$ 21,000	\$ 40,500
Storm window	\$ 120	30	\$ 3,600	20	\$ 620	\$ 4,220	\$ 1,400	\$ 5,000

Building envelope costs incur an additional equipment fee as this upgrade requires special services for blowing in insulation and demolition of materials. The added cost of adding basement ribbon insulation is included as well.

Table 17: Insulation cost estimate

	Unit Material & Equipment Cost	Quantity (ea.)	Total Material Cost	Labor Hours	Local Labor Cost	Local Total Cost	WMU Labor Cost	Total WMU Cost
Exterior façade demolition	\$ 15,000	1	\$ 15,000	120	\$ 6,000	\$ 21,000	\$ 8,400	\$ 23,400
New exterior insulation	\$ 15,000	1	\$ 15,000	120	\$ 6,000	\$ 21,000	\$ 8,400	\$ 23,400
Cedar siding	\$ 20,000	1	\$ 20,000	120	\$ 6,000	\$ 26,000	\$ 8,400	\$ 28,400
Sealants, barriers, fasteners, etc.	\$ 15,000	1	\$ 15,000	120	\$ 6,000	\$ 21,000	\$ 8,400	\$ 23,400
New double pane windows (refer to table)						\$ 27,600		\$ 40,500
<i>Options:</i>								
Interior insulation option	\$ 10,000	1	\$ 10,000	120	6000	\$ 16,000	\$ 8,400	\$ 18,400
Interior baseboard demo	\$ 10,000	1	\$ 10,000	120	6000	\$ 16,000	\$ 8,400	\$ 18,400
Basement/attic insulation	\$ 5,000	1	\$ 5,000	40	2000	\$ 7,000	\$ 2,800	\$ 7,800
New seal tight basement door	\$ 5,000	1	\$ 5,000	20	1000	\$ 6,000	\$ 1,400	\$ 6,400
Summarized Costs	Exterior Insulation Only					\$ 89,000		\$ 98,600
	Alt 4 Ext Ins + New Windows					\$ 116,600		\$ 139,100
	Alt 4-2 Interior Insulation Only					\$ 32,000		\$ 36,800

Energy cost savings were calculated using the energy simulation based on actual energy rates and the capital cost estimates. Energy cost results relating to each facility improvement category are shown in Table 18. Final recommendations are made in the next section. It is important to note that these estimates do not reflect actual contractor costs and are anticipated to increase due to abatements, unforeseen building issues, additional research data collection, and other complications.

Table 18: Facility improvement category options

Alternative Category	Alternative	Local Total Cost	WMU Total Cost	Yearly Total Operating Costs	Annual Savings vs As Is	Local PBT (years)	WMU PBT (years)
MEP Upgrades	Base upgrade	\$ 21,800	\$ 24,800	\$ 400	\$ 624	35	40
	Mini-split	\$ 23,600	\$ 26,800	\$ 480	\$ 544	43	49
Windows	Repairs	\$ 4,500	\$ 10,000	\$ 500	\$ 524	9	19
	Weatherstripping	\$ 900	\$ 3,100	\$ 510	\$ 514	2	6
	Weatherstrip & Film	\$ 1,500	\$ 4,800	\$ 430	\$ 594	3	8
	Refurbish	\$ 12,200	\$ 37,000	\$ 500	\$ 524	23	71
	Double Pane	\$ 27,600	\$ 40,500	\$ 340	\$ 684	40	59
	Storm Windows	\$ 4,220	\$ 5,000	\$ 440	\$ 584	7	9
Insulation	Exterior Insulation (incl. windows)	\$ 116,600	\$ 139,100	\$ 270	\$ 754	155	184
	Interior Insulation	\$ 32,000	\$ 36,800	\$ 280	\$ 744	43	49
Deep Retrofit	Mini-split, ext. ins., windows	\$ 140,200	\$ 165,900	\$ 200	\$ 824	170	201

The insulation, building envelope and airtightness target areas overlapped for solutions when reviewing all facility improvement options. Thus, the related alternative capital costs and energy savings are shown in Figure 37 graphically.

CAPITAL COST & ENERGY SAVINGS COMPARISON
Insulation, Building Envelope & Airtightness Options

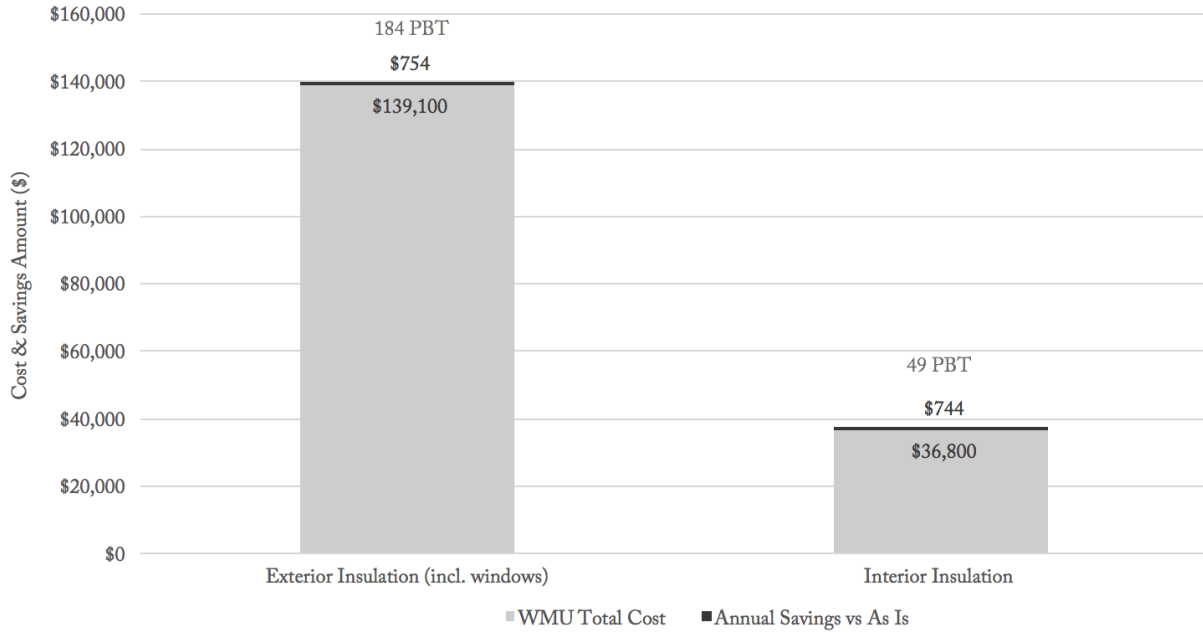


Figure 37: Insulation, building envelope & airtightness capital cost and energy savings

The six fenestration alternatives are compared graphically in Figure 38 to visually represent the amount of capital cost required for each and the sliver of annual cost savings resulting from each measure.

CAPITAL COST & ENERGY SAVINGS COMPARISON
Window Options

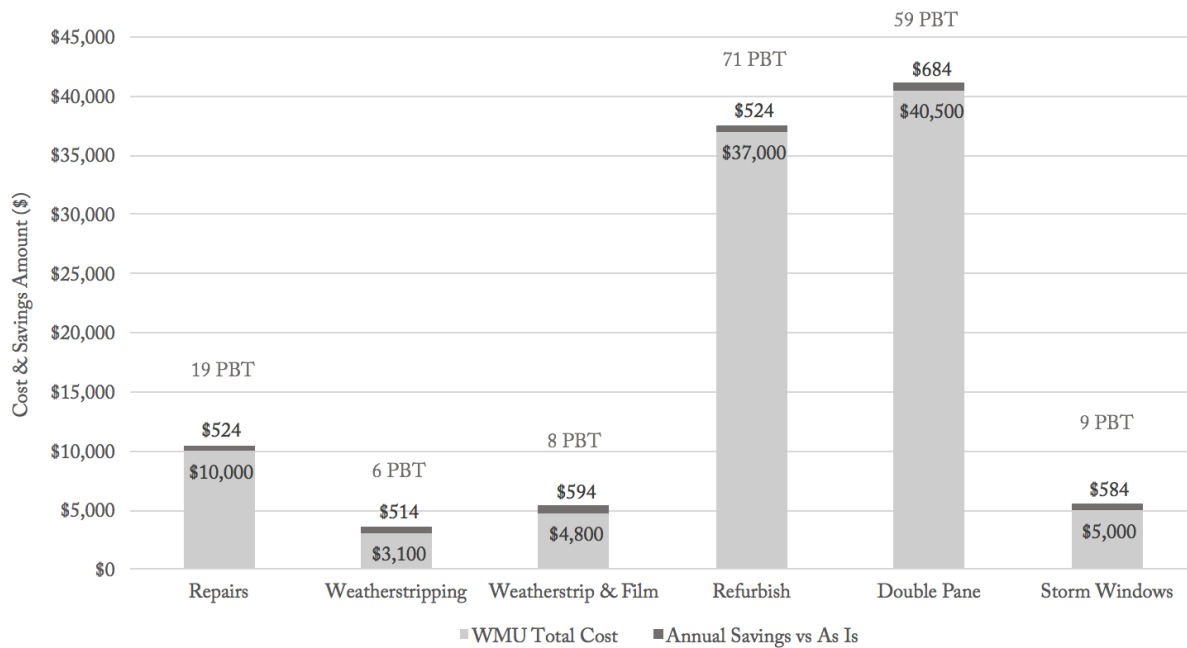


Figure 38: Fenestration capital cost and energy savings

The capital cost increase for implementing a mini-split system is demonstrated by the taller bar on the right. The base mechanical upgrade includes a replacement boiler, water heater, new ceiling and attic fans, new perimeter heat, and new LED lighting fixtures.

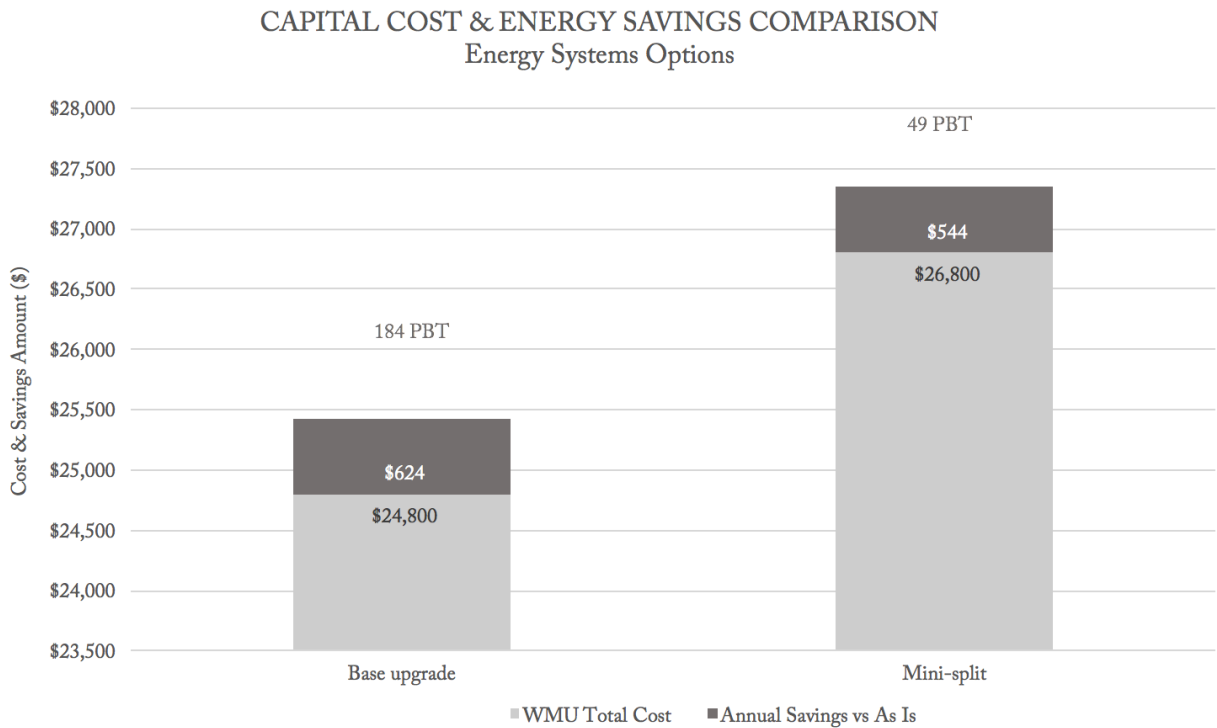


Figure 39: Energy systems capital cost and energy savings

Alternatives with new windows, deep retrofitting, new insulation, and new building envelopes are very costly with very long payback times. Alternatives with only window upgrades achieve lower energy savings and do not meet the Passive House standard, but are fairly inexpensive so result in rapid payback times.

Chapter 5

5 Conclusions and Recommendations

Historic renovations focusing on energy efficiency are known to be riddled with complications and intertwined issues. The Gibbs House is no exception. Deficiencies in each of the five Passive House target areas of insulation, fenestration, building envelope, airtightness, and energy systems were observed throughout various types of testing and simulation. The insulation analysis revealed deteriorated and nearly non-existent mineral wool insulation within the wall cavity. Original single pane glass windows have loose framework, broken panes, worn caulk, and a U-factor of 0.687 Btu/h-ft-F compared to new double pane U-factors below 0.31 Btu/h-ft-F. Balloon framing results in high heat flows specifically through air cavities and exterior siding. Infiltrations occurs at multiple points across the building envelope including the attic, basement, and floor joists and window framework is prone to air change rates greater than 2.8 ACH50 compared to the Passive House standard of 0.6 ACH50. Mechanical and plumbing equipment is out of date by at least eight years, lighting is non-LED with no occupancy sensors, and no mechanical air circulation is provided to occupants. Overall, the building was observed to not meet Passive House standards, as well as Michigan Building Code in some instances, in each target area.

However, none of these issues are irreparable. This study investigated facility improvement measures for each target area in order to improve occupant comfort and energy performance. The improvement measures are based on the alternative structure carried throughout the study and correspond to respective energy simulation results detailed throughout each Results section. Facility improvement measure recommendations for each target are summarized in Table 19 to highlight potentially cost effective solutions with lower capital costs than the complete deep retrofit. Refer to the Results section to review all possible options and resulting cost savings and payback times.

Table 19: Cost effective facility improvement alternatives

Alternative Category	Related Alternative	Core Issue	Improvement Recommendations	Capital Cost ¹	Energy Cost Savings	Simple PBT
Insulation	4-2	Deteriorated cavity insulation	Blown-in cellulose insulation to each cavity from the interior baseboards.	\$ 18,400	\$ 744.00	24.7
Building Envelope	4-2	Poor R-values	Insulate attic & basement ribbon	\$ 7,800	-	-
Airtightness	4-2	High infiltration rates at window framing	Seal basement and attic. New basement sealing door.	\$ 6,400.00	-	-
Fenestration	Alt 3F	Broken panes and framing. High overall U-factor	Repair broken panes and frames. Install exterior storm windows.	\$15,000.00	\$ 584.00	25.7
Energy Systems	Alt 2-2	Out-of-date equipment, inefficient lighting, low/poor air circulation	Install electric boiler, water heater, fin radiators, & mini-split. Install attic fan, ceiling fans, and LED lighting replacement. Include piping insulation & controls.	\$26,800.00	\$ 544.00	49.3
Final Recommendation Totals				\$74,400.00	\$ 1,872.00	40

¹ Incorporates updates in unconditioned spaces that may not have direct impact on energy savings, but will improve building condition and used spaces

Chapter 6

6 Future Work

The intent of the work is to provide analysis and recommendations for future facility improvements at the Gibbs House. Additional insight into actual contractor costs, creation of architect-drawn wall assembly detail drawings, and more formal coordination with WMU Campus Planning are required to move this work towards a renovation reality.

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Appendices

Appendix A: Gibbs House Asbestos and Lead Reports



WESTERN MICHIGAN UNIVERSITY

Environmental Health And Safety

Asbestos Report By Building

Gibbs House / Baker Farm

Sample	Room	Structure	Substrate	%Asb	Date	Description
7457	Bathroom, Grou	Ceiling	Plaster	nd	8-23-13	Ceiling Plaster, Surface Coat, White
7458	Bathroom, Grou	Ceiling	Plaster	nd	8-23-13	Ceiling Plaster, Base Coat, Gray, Cementious, Horsehair appears to be present
7848	Upstairs Bathroo	Wall	Plaster	nd	6-17-14	Plaster, Surface Coat, White
7849	Upstairs Bathroo	Wall	Plaster	nd	6-17-14	Plaster, Base Coat, Grey
7850	Upstairs Bathroo	Window	glazing	nd	6-17-14	Window Glazing, Gray, from "old window"
7851	Upstairs Bathroo	Floor	Linoleum	nd	6-17-14	Linoleum, Green with white/dark streaks
7852	Upstairs Bathroo	Floor	Tile	nd	6-17-14	9"x9" FT, Tan with green/yellow/brown streaks. Mastic=nd
8294	Exterior, Front D	Window	glazing	nd	9-15-16	Window Glazing, Gray, from Front door small rectangular windows

Lead Report by Building - Gibbs House

Building	Location	% Lead	Date	Color	Sample #
Gibbs House	solarium	26	8/6/2009	white wall paint	WM09-1592
Gibbs House	back porch	19	8/6/2009	white wall paint	WM09-1593
Gibbs House	stairway leading upstaris	nd	9/22/2010		WM10-1652
Gibbs House	Front Porch	24	6/11/2013	White	WM13-1760
Gibbs House	Front Porch	18	6/11/2013	light blue, interior	WM13-1761
Gibbs House	Upstairs bathroom	0.0028	6/13/2014	Light Green	WM14-1788
Gibbs House	Upstairs bathroom	0.022	6/13/2014	Cream	WM14-1789
Gibbs House	Upstairs bathroom	<RL	6/13/2014	White	WM14-1790
Gibbs House	Living room baseboard	positive	11/7/17	Cream	3M Lead Check Swab



Asbestos Analytical Laboratory Report
Laboratory ID # 101016

Western Michigan University
Attn: Jake Woods
1903 W. Michigan Ave
Kalamazoo, MI 49008
Mailstop 5485

Project: Gibbs House

Laboratory Report No: 17-2579B
Date Received by Lab: November 8, 2017
Date Samples Analyzed: November 8, 2017

The following samples have been analyzed for asbestos as requested. The results are compiled in the following table.

BDN Lab Number	Client Sample Number	Asbestos Identification (percent by weight)	Sample Description	Other Fibrous Materials	Binders or Fillers	Notes
17-12102	WMU17-8479	5% Chrysotile	Grey Solid	5% Cellulose	Calcite/Quartz	H
Analytical Method: EPA 600 / R-93/116						

Notes: H: Sample was homogenized, W: Sample was received wet, L: Sample was analyzed in layers, A: Sample was ashed to remove interferences and some organics may not be reported. *: other

The samples received by this laboratory will be archived for 30 days and then destroyed. The client may request longer archival or sample return for a nominal fee. This laboratory is in compliance with the QC/QA requirements described in the method and participates in the AIHA PAT Bulk Asbestos Program. This laboratory is currently rated as Proficient. Please contact me at (269) 329-1237 if you have any questions. It has been a pleasure assisting you.

Travis Noa
Project Manager
BDN Industrial Hygiene Consultants, Inc.

CREATING SAFE WORK

8105 Valleywood Lane • Portage, MI 49024 • 269 329-1237 • www.bdnhc.com



Asbestos Bulk Chain of Custody

SEND TO: Attn: Brent Bassett
 BDN Industrial Hygiene Consultants, Inc.
 8105 Valleywood Ln.
 Portage, MI 49024
 Phone 269.329.1237 Fax 269.329.7446

Client Job Number: _____

Client Name:	<u>Western Michigan University</u>	Phone:	<u>269-387-5589</u>
Address:	<u>1903 W. Michigan Ave.</u>	Fax:	<u>N/A</u>
	<u>Kalamazoo, MI 49008</u>	Project Site:	<u>Gibbs House</u>
	<u>Mailstop: 5485</u>	Contact Person:	<u>Jake Woods</u>
		Email:	<u>jacob.woods@wmich.edu</u>

Sample #	Sample Description	Location Sampled
WM17-8479	Window Glazing, Gray	Gibbs House, Front Exterior Windows, Small Rectangular Panes

Date to Lab: 11-8-17 1 BUSINESS DAY (24 Hours) Up to 5 BUSINESS DAYS
 Total # Samples: 1
 Relinquished by: [Signature] Date: 11-8-17 Time: 1:06
 Received by: [Signature] Date: 11/8/17 Time: _____

Comments: _____
How do you want to receive results? Call Fax Mail Email
 BDN Use Only Cash Check Credit Card P.O. # _____
 Total Amount \$ _____

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Appendix B: Mathcad Two Dimensional Results

2D Thermal Conduction
Gibbs Exterior Wall

$$T_{\text{infi}} := 62.3 \quad A := 2924\text{ft}^2 \quad \text{wall surface area}$$

$$T_{\text{infe}} := 19.3$$

$$y := 5 \quad y := 5 \quad \text{use y.dot for integral}$$

$$f_3(y) := (22.898 + 0.7649 \cdot y) \quad \text{exterior temp distribution}$$

$$f_4(y) := (61.1 + 0.393 \cdot y) \quad \text{interior temp distribution}$$

Wall Thicknesses:

material	thickness	x location	thermal conductivity
plaster	$x_{\text{pl}} := 0.0833$	$x_1 := 0\text{ft}$	$k_1 := 0.25$
insulation	$x_{\text{ins}} := 0.33$	$x_2 := x_{\text{pl}}$	$k_2 := 0.00676$
rigid insulation	$x_{\text{rig}} := 0.125$	$x_3 := x_2 + x_{\text{ins}}$	$k_3 := 0.00367$
air gap	$x_{\text{gap}} := 0.04165$	$x_4 := x_3 + x_{\text{rig}}$	$k_4 := 0.0149$
aluminum siding	$x_{\text{al}} := 0.0833$	$x_5 := x_4 + x_{\text{gap}}$	$k_5 := 121$
		$x_6 := x_5 + x_{\text{al}} = 0.663$	

$$\text{overall geometry} \quad a := 0.663 \quad b := 10$$

$$n := 1 \quad \text{Taylor Series converges early}$$

$$f_4(5) = 63.065$$

$$T_1(x, y) := \frac{\frac{2}{b} \sum_{n=1}^3 \left[\frac{\sinh\left(\frac{n\pi}{b}\right)(a-x) \sinh\left(\frac{n\pi}{b}\right)y}{\sin\left(\frac{n\pi}{b}\right)a} \right] \int_0^b f_4(y) \sin\left(\frac{n\pi}{b}\right)y dy - T_{\text{infe}}}{T_{\text{infi}} - T_{\text{infe}}}$$

$$T_1(x, y) = 58.141$$

$$T_e(x, y) := \frac{\frac{2}{b} \sum_{n=1}^3 \left(\frac{\sinh\left(\frac{n\pi}{b}\right) \cdot x \sinh\left(\frac{n\pi}{b}\right) \cdot y}{\sin\left(\frac{n\pi}{b}\right) \cdot a} \right) \int_0^b f_3(y) \sin\left(\frac{n\pi}{b}\right) \cdot y \, dy - T_{infe}}{T_{infi} - T_{infe}}$$

$$T_e(x_6, 5) = 25.304$$

$$T(x, y) := T_i(0, 5) + T_e(x_6, 5) \quad \text{for entire cross section}$$

Redefine variables with units

For first section:

$$T(x, y) := T_i(0, 5) + T_e(x_1, 5)$$

$$T_i(x, 5) = 58.141$$

$$T_e(x_2, 5) = 2.786$$

$$T_e(x_3, 5) = 15.599$$

$$T_e(x_4, 5) = 20.452$$

$$T_e(x_5, 5) = 22.07$$

$$T_e(x_6, 5) = 25.304$$

Temp distributions for each layer:

lengths w/ units

thermal conductivity w/ units

$$T_1 := T_i(0, 5) + T_e(x_2, 5) = 60.926$$

$$x_{pl} := 0.0833 \text{ ft}$$

$$k_1 := 0.25 \frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot \Delta^\circ\text{F}}$$

$$T_2 := T_i(0, 5) + T_e(x_3, 5) = 73.74$$

$$x_{ins} := 0.33 \text{ ft}$$

$$k_2 := 0.00676 \frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot \Delta^\circ\text{F}}$$

$$T_3 := T_i(0, 5) + T_e(x_4, 5) = 78.593$$

$$x_{rig} := 0.125 \text{ ft}$$

$$k_3 := 0.00367 \frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot \Delta^\circ\text{F}}$$

$$T_4 := T_i(0, 5) + T_e(x_5, 5) = 80.21$$

$$x_{gap} := 0.04165 \text{ ft}$$

$$k_4 := 0.0149 \frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot \Delta^\circ\text{F}}$$

$$T_5 := T_i(0, 5) + T_e(x_6, 5) = 83.445$$

$$x_{al} := 0.0833 \text{ ft}$$

$$k_5 := 121 \frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot \Delta^\circ\text{F}}$$

$$x_w := x_{pl}$$

$$x_{2v} := x_1 + x_{ins}$$

$$x_{3v} := x_2 + x_{rig}$$

$$x_{4v} := x_3 + x_{gap}$$

$$x_{5v} := x_4 + x_{al}$$

Temp dist rewritten with units

$$\begin{aligned}T_{11} &:= 60.926\text{ }^\circ\text{F} & T_{44} &:= 80.21\text{ }^\circ\text{F} \\T_{22} &:= 73.74\text{ }^\circ\text{F} & T_{55} &:= 83.445\text{ }^\circ\text{F} \\T_{33} &:= 78.593\text{ }^\circ\text{F}\end{aligned}$$

Heat flow thru each layer

$$q_1 := k_1 \cdot A \cdot \frac{T_{11}}{x_1} = 1.269 \times 10^3 \frac{1}{\text{s}} \text{ Btu}$$

$$q_2 := k_2 \cdot A \cdot \frac{T_{22}}{x_2} = 7.086 \frac{1}{\text{s}} \text{ Btu}$$

$$q_3 := k_3 \cdot A \cdot \frac{T_{33}}{x_3} = 2.981 \frac{1}{\text{s}} \text{ Btu}$$

$$q_4 := k_4 \cdot A \cdot \frac{T_{44}}{x_4} = 11.266 \frac{1}{\text{s}} \text{ Btu}$$

$$q_5 := k_5 \cdot A \cdot \frac{T_{55}}{x_5} = 8.048 \times 10^4 \frac{1}{\text{s}} \text{ Btu}$$

total wall heat flow:

$$q_{\text{tot}} := q_1 + q_2 + q_3 + q_4 + q_5 = 8.177 \times 10^4 \frac{1}{\text{s}} \text{ Btu}$$